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Pulse tube measurement of bulk modulus of visco-elastic composite materials: theory and practice

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A method for the determination of the bulk moduli and loss factors of micro-voided composite materials is presented. The method requires that the reflection and transmission coefficients of a tile of uniform thickness are determined in both amplitude and phase as functions of frequency. Reduction to the bulk modulus and loss factor then proceeds by using the analytic properties of a function of a complex variable derived from the reflection and transmission coefficients. A pulse tube is used for the determination of the complex reflection and transmission coefficients. Although other measurement techniques are available, the pulse tube has proved to be versatile in covering a large range of the frequency-temperature master curve for typical composite materials used in underwater acoustics. It achieves this versatility by using an anti-freeze/water mixture as the medium following which measurements can be made over a range of different temperatures.

1 Introduction

Composite visco-elastic materials consisting of micro-inclusions embedded in a visco-elastic substrate have found considerable application in underwater acoustics by virtue of the damping properties of their compression waves. These damping properties arise through boundary interaction between substrate and inclusions conveying the damping inherent in the substrate shear modulus onto compression. While many theoretical models exist for the prediction of composite properties (e.g. [1, 2] where reference is made to other models), unresolved issues remain before having full confidence in them. Therefore the capability of measurement of the macroscopic properties of these materials remains an important part of noise reduction programmes.

We present here a method of measurement enabling the deduction of bulk modulus over a critical part of the frequency-temperature master curve [3]. Practical aspects of the method are presented in §2, but in outline it is based on determining complex reflection and transmission coefficients of samples placed in a pulse tube. Complementary to the measurements, a method of reduction of these data is given in §3 using analyticity properties of certain complex functions in an interesting and illuminative way.

The reduction method is unlikely to be new, but appears not to have received wide dissemination, probably for the reason that panel transmission characteristics are of primary interest, not the elastic properties that give rise to them [4, 5]. However, in our case knowledge of the elastic properties of these materials is central to the operation of validated computer models, e.g. FLAAPM [6], for the properties of multi-layered baffles. This makes the reduction method worthy of further exposition for the determination of these difficult to obtain properties.

Results of analysis of real data are presented in §4 and limitations and potential problems are explored further in §5. In particular, measurement data at different temperatures at constant static pressures greater than ambient do not give contiguous sections of the master curve.

2 Data collection and processing

The requirement for the pulse tube is to provide reflection and transmission coefficients of a sample of a circular disc of a test material with parallel faces, these coefficients to be taken as representative of an infinite plane sheet of material. The tube is a water-filled thick walled cylinder with a ball projector hydrophone fitted into the bottom end cap, the top end cap providing access to the water column. An insert of high transmission loss material is fitted below this top end plate to reduce radiation from the end cap due to compression waves in the tube wall.

The water column and sample can be subjected to hydrostatic pressures up to 40 bars and the temperature controlled between 0 and 35°C. Hydrophone inserts in the tube allow hydrophones to be inserted into the water column above and below the sample disc and are therefore available to measure pressure on the reflected or transmitted side of the sample. The instrumental set-up is arranged to excite the tube with a burst chirp.

The useful frequency range is ~400Hz to 12kHz, the lower limit determined by the performance of the projector and the higher by the frequency of the first cut-on mode in the water column. However, for test samples having sound speeds lower than water, the high frequency limit may be significantly reduced as will be demonstrated.

Frequency response functions between projector source signal and hydrophone outputs are measured using an HP analyser and are then transformed into the impulse response domain. Here, unwanted reflections are gated out and then re-transformed to the frequency domain. Reflection and transmission coefficients with phase follow straightforwardly. It is noted that the gating process reduces frequency resolution of the computed coefficients to ~250Hz, but this is not critical provided the frequency dependence of the effective material properties is relatively smooth.

3 Theory

The key to deducing bulk modulus of a layer of material from its reflection and transmission coefficients lies in determination of the complex propagation parameter, p, across its thickness, L, defined by

$$p = e^{i\omega L/c} = e^{ikL} \qquad (1)$$

encapsulating phase shift *and* attenuation arising from the complex wavenumber $k = \omega / c$ of a plane compression wave over a distance L. ω is angular frequency and c the unknown sound speed in the sample material. Standard analyses for propagation through layered media (see [7] for example) yield the following for the propagation parameter:

$$p = z \pm \sqrt{z^2 - 1} \quad \text{where} \quad z = (1 - R^2 + T^2)/2T \qquad (2)$$

Here, R and T, are the *complex* reflection and transmission coefficients. Note that (2) gives two possible values for complex p and a rationale is required for selecting the physically correct one. The obvious rationale of selecting the root for which $|p|<1$ would be satisfactory given *perfect error-free* R and T, but it can fail if R and T are subject to experimental error. In light of this an alternative rationale has been sought and found. It is based a principle that the locus of the propagation parameter in the complex plane as frequency changes should be continuous and lie on the correct sheet of the Riemann surface. It can be shown that, for error-free R and T, the following factorisation of (2) automatically satisfies this

$$p(z) = z - \sqrt{z+1}.\sqrt{z-1} \qquad (3)$$

given the convention that the branch cut for complex square root is the negative real axis. Equation (3) then has a single branch cut on the real axis joining the two branch points at ±1. The wavenumber within the sample, hence compression wave speed follows from equation (1):

$$k = (ln(p)+2\pi in)/iL \qquad (5)$$

The integer n is the number of whole wavelengths λ across the thickness L *plus* $\lambda/2$ and reflects the fact that the complex log function is multi-valued. Again, as p is a continuous function of z, k is also continuous function of p and is accommodated if n is incremented by 1 for each *anticlockwise* crossing of the negative real axis of the complex p-plane and reduced by 1 for each clockwise crossing.

To illustrate this rationale, the theoretical reflection and transmission coefficients for a layer of known properties are known (see [7]). Thus, Figs.1 shows the loci with respect to increasing frequency for the parameters z and p for these idealised R and T of a layer for which a bulk modulus (with damping) and density have been assigned.

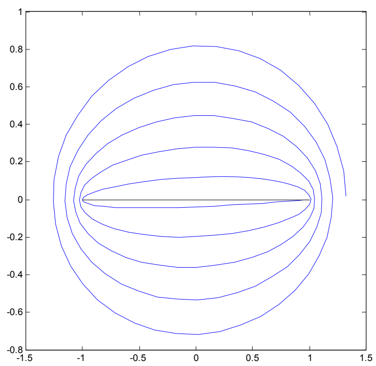

Fig.1 Idealised locus of the z parameter.

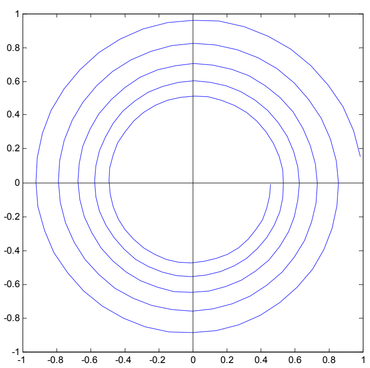

Fig. 2 Idealised locus of the p parameter.

Starting at the +1 branch point for zero frequency, the z-locus winds its way around the branch cut in a *clockwise* direction with increasing frequency and does not cross the cut, as it should not in this theoretical demonstration. Correspondingly, p, spirals inwards in an anticlockwise sense starting from the positive real axis, the inward spiral is indicative of evanescence of the damped wave across the sample: without damping the locus would remain on the unit circle. The compression wavenumber follows from equation (5) with n being incremented in the manner already described.

4 Analysis of real data

As the first example of real pulse tube data we simply take the water column between the 'reflection' and 'transmission' hydrophones as a fundamental layer sample with no actual sample in place. In this case reflection coefficient is zero and the transmission coefficient is as deduced from the transfer function between the two hydrophones. Thus Fig.3 shows the z-locus for 1.4m of water column between the hydrophones.

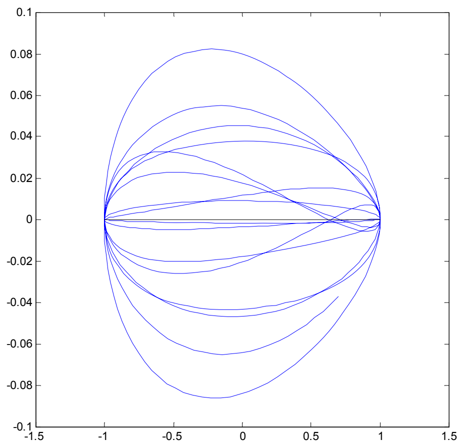

Fig. 3 Locus of z-parameter for the water column between hydrophones

Clearly, the locus has several crossings of the branch cut between ±1, but the excursion into the wrong sheet of the Riemann surface is not great, being at most .03 and easily accommodated by taking the opposite sign (i.e. '+') against the square roots of equation (3) for the duration of such excursions.

The resulting unwrapped wavenumber-frequency diagram is as in Fig. 4. Note that the small excursions onto the wrong Riemann sheet has resulted an insignificant negative imaginary component to wavenumber (magnified x100 in the figure) as indeed is the positive imaginary component. From the figure we can deduce the fluid wave speed as 1550 m/s.

Before examining the reduced real data from the pulse tube with a sample present, one further element is added to the reduction process. This follows a pertinent discussion with Prof. V. Humphreys who noted that R and T are not independent and we need not retain the *measured* values of both R and T. Now the one difficult measurement to obtain with accuracy is sample position. This did not matter for T as phase adjustment for transmission requires only the water column length between hydrophones, independent of sample position.

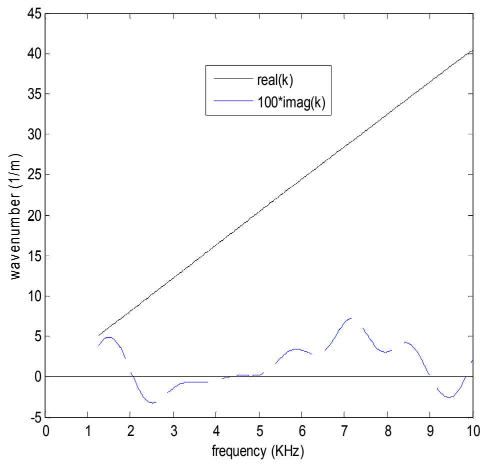

Fig.4. Unwrapped wavenumber-frequency diagram.

In contrast phase adjustment for R is not sample position independent and therefore carries the inherent inaccuracy of sample position. However, Prof. Humphrey's observation means that we can relinquish the inaccurately *measured* reflection coefficient, after its initial and use

$$R = \frac{(\alpha^2 - 1)(1 - p^2)}{4p\alpha}t \qquad (9)$$

iteratively updating R and p in turn. In (9), $\alpha=(\rho_2 c_2)/(\rho c)$ is the ratio of sample and fluid densities and sound speeds. The iteration procedure is rapidly convergent.

The sample considered here is taken from one of many composite materials used in underwater acoustics as described in [5] for example. Measurements were made at 3 temperatures namely 0°C, 4°C and 10°C and at 4 static pressures of $.08P_{ref}$, $.58P_{ref}$, $.8P_{ref}$ and P_{ref}.

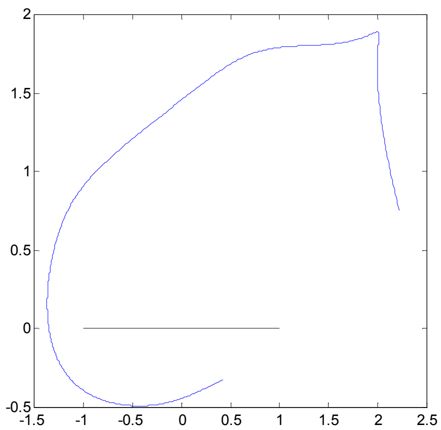

Fig.5 z-parameter locus for the composite sample.

Fig. 5 shows the *z*-locus for the 10°C case and Fig. 6 the corresponding *p*-locus over a frequency range 1.25 to 10 KHz. It is evident that, while the loci behave as expected over the greater part of their lengths, a kink occurs in each at the same frequency of 7.925 KHz. This behaviour is indicative of a higher order wave-guide cut-on in the sample. At higher frequencies, the data will be contaminated by the presence of this mode and we have no current prescription to handle it. Indeed the data will suffer contamination before that frequency as even the approach to cut-on will be characterised by reduction in evanescence across the sample thickness of the pre-nascent mode. Thus we discount data for the determination of bulk modulus and loss factor for frequencies over 6.25 KHz.

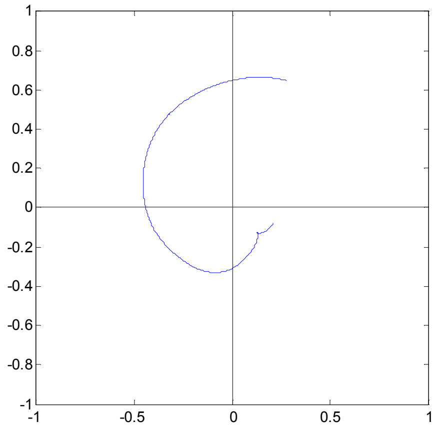

Fig. 6 p-locus of the composite sample.

Fig. 7 shows bulk modulus and Fig. 8 the loss factor of the sample as deduced from measurements made at the 4 pressures of $.08P_{ref}$, $.58P_{ref}$, $.8P_{ref}$ and P_{ref} and three temperatures 0°C, 4°C and 10°C.

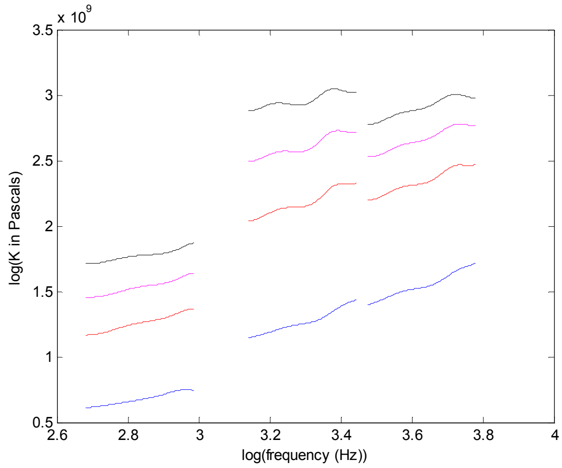

Fig. 7 Bulk modulus for 4 static pressures and 3 temperatures.

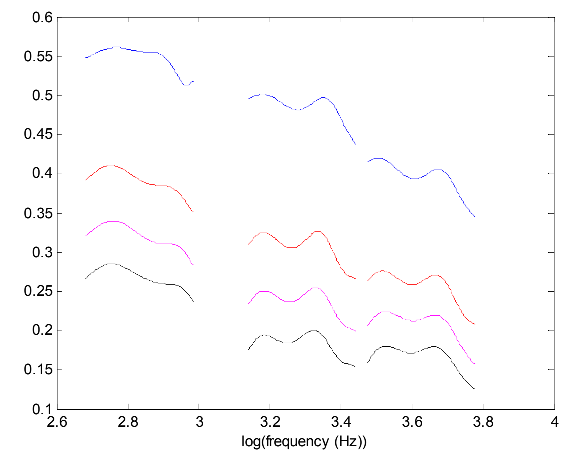

Fig. 8 Loss factor for 4 static pressures and three temperatures.

There are three things to note. Firstly the abscissa represents log(frequency) transformed to a master temperature of t_s (=0°C in this case) according to the WLF transform [3] given by

$$\Delta\{\log(\mathit{frequency})\} = \frac{-8.86(t - t_s)}{101.6 + (t - t_s)} \qquad (11)$$

with the right-hand column of plots in both figures being from the 0°C data, the centre from the 4°C data and the left-hand from the 10°C data. Secondly, bulk moduli are monotone increasing with pressure and loss factors monotone decreasing. This is entirely to be expected, as the compressible inclusions will progressively collapse with increasing pressure making them less effective in: (a) reducing the effective bulk modulus; (b) making the intrinsic lossiness of shear modulus transfer onto compression waves.

Thirdly, it is apparent that, except at the lowest pressure (.08P_{ref}), the three different temperature segments of bulk moduli do not fall satisfactorily on to a master curve. Indeed, with increasing pressure the three segments seem to drift their own particular way! Perhaps surprisingly the results for loss factors do seem to fall on a master curve for all pressures. This is not understood at present, but these issues are discussed more fully in the next section.

5 Limitations and issues arising

No measurement technique or reduction method will be error/problem-free and the method here is no exception, the first limitation being that of a limited frequency range for valid data. The low frequency limit is <~1KHz and the upper limit dictated by the frequency of the first cut-on wave-guide mode *in the material sample*. Thus this upper limit is dependant on the wave speed of sample being studied. To overcome these limitations and extend the part of the master curve that can be reached by the measurements, data was obtained at different operating temperatures. This process worked well for close to zero static pressure, but suffered from the defect that the results obtained at different temperatures were no longer contiguous with respect to each other when reduced to the master temperature. The reason for this (affirmed by Paul Howgate, IMT Sweden; private communication) is that the WLF transform works for constant static *strain*, not constant stress, but the results presented here were for constant static stress for which the sample being measured would adopt *different static strains at different temperatures.* Resolution of this would require carrying out measurements at different pressures for different temperatures that would give the same strain state. The present authors do not know the full resolution of this problem since a further issue arises: at colder temperatures the test samples may take a long time to equilibrate and in Paul Howgate's words: 'If you have a couple of millennia to spend this is O.K.! However, we mere mortals prefer to get the testing over with in a reasonable time scale!'

References

[1] PR Brazier-Smith. "A Unified Model for the Properties of Composite Materials" IUTAM Symposium on Diffraction and Scattering in Fluid Mechanics and Elasticity. ID Abrahams, PA Martin and MJ Simon (Eds.) Kluwer Academic Publishers. (2000).

[2] WH Parnell. "Homogenization Techniques for Wave Propagation in Composite Materials" PhD Thesis. Manchester. (2004).

[3] JD Ferry. "Visco-elastic properties of polymers" Publisher J. Wiley & Sons Inc." (1980)

[4] VF Humphrey and HO Berktay, "The Transmission Coefficient of a Panel Measured With a Parametric Source" J. Sound Vib. 101(1) 85-106 (19**)

[5] FH Kerr and AM Baird. "Acoustic Modelling of Signature Reduction Materials for Underwater Applications" IUTAM Symposium on Diffraction and Scattering in Fluid Mechanics and Elasticity. ID Abrahams, PA Martin and MJ Simon (Eds.) Kluwer Academic Publishers. (2000)

[6] CP Walker and PA Cotterill, "A Model for the Acoustic Response of a Fluid-Loaded Cylinder with Variable Internal Stiffeners" Proceedings of the Institute of Acoustics. Vol. 27 (2005)

[7] LM Brekhovskikh, "Waves in Layered Media" Academic Press. (1980)