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surface elastic waves in granular media under gravity and their relation to booming avalanches

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Due to the non-linearity of Hertzian contacts, the speed of sound in granular matter increases with pressure. Under gravity, the non-linear elastic description predicts that acoustic propagation is only possible through surface modes, called Rayleigh-Hertz modes and guided by the index gradient. Here we directly evidence these modes in a controlled laboratory experiment and use them to probe the elastic properties of a granular packing under vanishing confining pressure. The shape and the dispersion relation of both transverse and sagittal modes are compared to the prediction of non-linear elasticity that includes finite size effects. This allows to test the existence of a shear stiffness anomaly close to the jamming transition.

1 Introduction

The elasticity of a granular material is supported by the elastic inter-grain contact network. The force required to bend the contact between two spheric grains is known as Hertz contact force and varies as $F \propto \delta^{3/2}$. It acts as a non-linear spring because of spherical geometry of grains. Taking into account this nonlinearity and considering the granular medium as a continous one trough a meanfield calculation, it can be demonstrated that waves velocity $c \propto P^{1/6}$. Under gravity, it appears a pressure gradient, $P = \rho g z$, thus $c \propto z^{1/6}$. Because of increase of c with the depth, acoustics waves are refracted to the surface by the index gradient and such a system cannot propagate bulk waves, the only possibility for propagating acoustic waves is surface waves[1],[2]. Here we present an experiment conceived to evidence these surface waves [3].

2 Experimental Setup

In order to explore surface waves properties, a laboratory experiment was built. It is a channel of $20cm$ square cross-section by a lenght of $1.40m$. It is filled by glass beads of 150 μm average diameter. The support of the experiment lays on acoustic damping bloc to avoid parasite vibrations. Two kinds of excitating devices are used, one for saggital waves generation and the other for transverse waves. Both are electromagnetic transducers. The first is a coil mounted on ball-bearing carriage, guided on an axis, moving in the field of a permanent magnet (1a). The carriage is connected to the granular packing through a very rigid transverse metalic blade . This feature allows to disable elastic coupling from the transducer and to prevent from interfering signal while longitudinal wave generation. The second, used for transverse waves excitation is a small magnetic rod placed in the magnetic field of a coil, communicating its inertia to the body of vibrating device using linear springs (1b). Because we expect a multimode emission [2], the signal emitted is a gaussian wavepacket (1c).

Signal measurements are recorded using accelerometers buried in the granular medium. Jia[4] reported that for observing a coherent signal corresponding to which is propagating in the effective medium, sensors which size is big for grain size must be choosen. Thus, we took care to separate the scales of grains, sensors (accelerometer size is $13mm$diameter by $15mm$ high), wavelength and experiment dimensions. Reader can see two samples of signal(1d-e) recorded in the granular packing and notice the coherent part and its quality.

On (1d), the packing was prepared by sweeping a ruler longitudinally and transversally to remove memory effects of the filling. On (1e), the medium was compacted between the emitter and the sensor. We can notice that the phase and the center of wavepacket are sensitive to the history of the granular packing. The first preparation process was choosen because it allows to perform reproducible experiments.

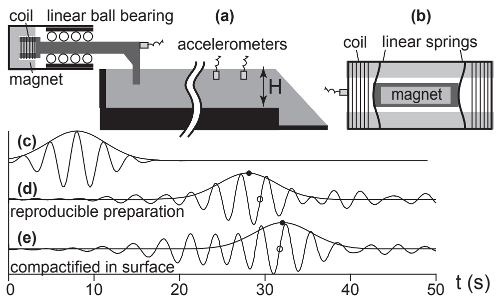

Figure 1: Experimental setup : **(a)**The channel and the saggital waves emitter **(b)**Transverse waves emitter **(c)**Saggital signal recorded on the source by an accelerometer **(d-e)** Signal recorded in the channel at $60cm$ from the source for two different granular packings Note the shift of the wave-packet centre (•) and of the phase (○).

3 Results

3.1 Propagating waves

The first experiment consist of checking if we indeed can propagate waves at the packing surface. Accelerometers were moved from the neighbourhood of the emitter along the channel, in order to measure time of flight and phase shift as function of the position (2). It shows clearly the propagation of a coherent wavepacket with a small distorsion of the signal. For sagittal waves, at the frequency of $315Hz$, the phase velocity measured is $71.3 \pm 0.3m/s$ and the group velocity is $31.2 \pm 0.2m/s$

3.2 Dispersion relation

Next, we determine the dispersion relation for glass-beads packing. The sensors were put at fixed positions and measurements were made for a range of frequencies spanning from $300Hz$ to $700Hz$ by $2Hz$ increments, for the saggital waves and from $300Hz$ to $550Hz$ by $3Hz$ increments, for the transverse waves. The results

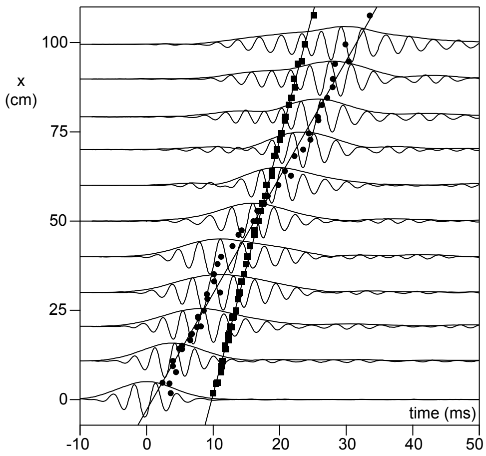

Figure 2: Space-time diagram. It shows the propagation of a gaussian wavepacket along the channel.The time of flight (•) is is located by the maximum of the enveloppe. Then signal's phase (■) is determined by fitting the signal with a gaussian wavepacket around the maximum

are ploted on graph(3) and compared with the theoretical prediction derived from [2]. It can be seen that the experimental data and the theoretical curves match pretty well. At low frequencies, the theoretical displays a cut-off frequency, this is the typical behavior of a waveguide.

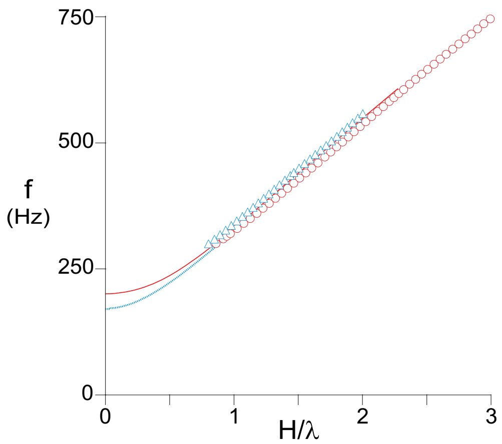

Figure 3: The experimental dispersion relation (symbols) are superimposed on theoretical curves (lines) for both sagittal waves (red,○) and transverses (blue,△). Frequency F is ploted as function of H/λ where H is the depht of the channel and λ the wavelength

Thus the channel design allows to generate a propagating monomode signal which is analysed in order to extract the corresponding dispersion relation.

3.3 Surface waves

Finally, we fixed the frequency and the position of the accelerometer along the axis of the channel. We measured the amplitude and the phase of the signal versus the depht of the sensor. Results are displayed on (4).

(4) shows both the vertical and the horizontal components are decreasing with depth and the penetration length is about one half of the wavelength. Moreover for the vertical component U_z the phase remains constant.The longitudinal component U_x has the same feature but it presents a phase jump that can be interpretted as a vibration node, as expected from the theoretical mode's shape. The annulation of the amplitude on a depth comparable to wavelength and the constant phase as function of the depth are the features of surface waves and confirms the monomode signal generation.

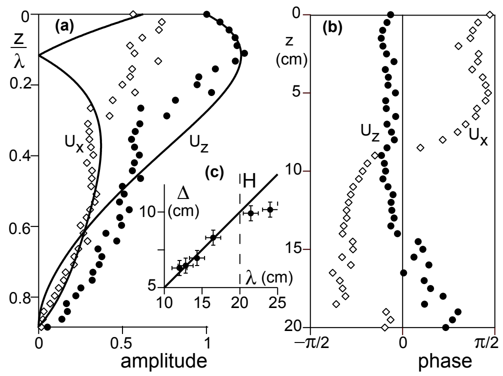

Figure 4: (a) Amplitude as function of depht z/λ; (b)phase as function of depht z/λ. (•)represents the horizontal displacement of saggital waves, (◇) is the vertical component. The solid curves represents the theoretical shape of propagating modes. (c) inset : penetration depht vs wavelength.

3.4 Discussion

The ratio v_g/v_ϕ provides an information on the nonlinearity of the contact force. The description of elasticity using Hertz-contact force yields the limiting value $v_g/v_\phi = 5/6$ in the case of a half-infinite medium. The experimental ratio v_g/v_ϕ is displayed on (5)

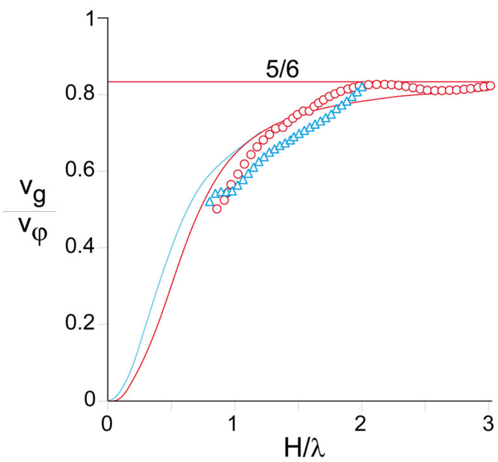

Figure 5: The ratio v_g/v_ϕ as function of H/λ.Symbols represents experimental data and lines the theoretical predictions. (Sagittal waves : red,○; transverses : blue,△)

It can be seen that for large H/λ, v_g/v_ϕ tends indeed toward the Hertz scaling value 5/6. For $H/\lambda < 1.5$ Both the theoritical curve and the experimental data go to zero as expected for the waveguide cut-off frequency.

A second very important fact can be pointed out concerning the value of the shear modulus. In [2] the dispersion relation for the surface modes was derived, using a meanfield elastic model consistent with Hertz force law [5],its expression yields :

$$f = \alpha \mathcal{A}^{1/2} \left(\mathcal{B} + \frac{5\mathcal{A}}{3} \right)^{-1/6} \left(\frac{E}{\rho} \right)^{1/3} g^{1/6} \lambda^{-5/6} \qquad (1)$$

where E is the elastic modulus of material, $\mathcal{A}$ is a dimensionless coefficient describing the shear and $\mathcal{B}$ is a dimesionless coefficient describing compression of the granular material [5], in the limit where $\lambda << H$. For our experiments both transverse and saggital modes dispersions are nearly equals. In such a case the theory predicts that $\mathcal{A} < \mathcal{B}$. It indicates that the restoring process for the force balance is mainly the shear elasticity. In that case, dispersion relation can be written :

$$f = 0.77 \mathcal{A}^{1/2} \mathcal{B}^{-1/6} \left(\frac{E}{\rho} \right)^{1/3} g^{1/6} \lambda^{-5/6} \qquad (2)$$

For experimental datas, where the curve $v_g/v_{\phi)}$ reaches 5/6, $f\lambda^{5/6} \approx 77 \pm 1 m^{5/6} s^1$. It corresponds to $\mathcal{A}^{1/2}\mathcal{B}^{-1/6} \approx$ 0.23, whereas the mean-field expectation is 0.40 for frictionless grains and 0.61 for infinite friction. Thus the shear modulus is 3 to 5 times smaller than expected by mean-field theories. This abnormality is coherent with numerical simulations [6]. The "soft-modes" theory can explain this mean-field failure.

4 Conclusion

We have evidenced that in a granular packing under gravity, we can observe surface waves propagation by exciting them in two different ways : saggital and transverse. Using the waveguide feature of a glass-beads filled channel, we manage to isolate the fondamental mode of propagation and show that the dispersion relation corresponds to the Rayleigh-Hertz waves as described in [2]. However in spite of the [7], we find that Hertz scaling is recovered. We quantitatively measure an elastic coupling that seems to correspond to a very soft shear stiffness.

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