On a new approach to numerical modeling of a low-frequency underwater sound in 2 and 3-dimensional oceanic waveguides

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A new method to underwater sound field calculations is proposed in application to irregular in horizontal plane waveguides. It realizes the full two-way coupled-mode approach and exploits an idea of a problem solution dependence on the certain variable parameter that is the position of a boundary of the irregular region. With respect to this parameter for waveguide modes, an initial value problem can be formulated in horizontal plane that is equivalent to the boundary value problem for the original acoustic wave equations. This fact allows simulating sound field in waveguides based on the evolution ordinary differential equations without traditional approximations and so for arbitrary both source distance from the irregular region and degree of irregularities. Examples of a simulation for two dimensional irregular waveguide models with upslope rigid and absorbing penetrable bottom are presented for low frequencies and shallow sea conditions. They illustrate the serious difference between the exact and approximate solutions that appears due to both strong modes coupling and backscattering within the considered irregular waveguides.

1 Introduction

It is well known that regularities of acoustic field propagation in layered and nonlayered oceanic waveguides directly correlate with horizontal heterogeneities. If the heterogeneities are sufficiently smooth, very often no qualitative differences are observed in the laws of field formation compared to the layered situations. Then in order to make quantitative estimates of the losses during acoustic wave propagation, it is possible to limit ourselves to the framework of known methods, such as the adiabatic approximation, Wentzel-Kramers-Brillouin (WKB), or parabolic equation (PE) method [1, 2]. Serious difficulties arise when distortions appear in the smooth character of horizontal irregularities in the medium. In this case, the problem in the horizontal plane becomes a boundary problem and the approximate methods mentioned above are of little use. There are only a few works in acoustic literature in which this problem is considered in the exact formulation, and their authors nevertheless do not avoid approximations at different stages of the solution [3, 4]. If heterogeneities are three dimensional (3D) there are no related works. In this paper, the new approach to simulate such problems is presented. It is based on the first-order equations with initial conditions, which provide exact descriptions of the acoustic situation in the cases when distortions appear in the smooth character of horizontal heterogeneities in the medium as well as in the other applicability conditions of the known approximate methods.

2 Problem statement, governing equations

Let us address to original linear acoustic equations for the fields of tonal point source in 3D heterogeneous ocean. In cylindrical coordinates (r, z, ψ) there is the following boundary problem:

$$\frac{\partial}{\partial z} p(r,r_0,z,z_0,\psi,\psi_0) - i\omega\rho(r,z,\psi)\, w(r,r_0,z,z_0,\psi,\psi_0) = 0\,,$$

$$\frac{\partial}{\partial r} p(r,r_0,z,z_0,\psi,\psi_0) - i\omega\rho(r,z,\psi)\, u(r,r_0,z,z_0,\psi,\psi_0) = 0\,,$$

$$\rho\{\frac{\partial}{\partial z} w + [\frac{\partial}{\partial r} + \frac{1}{r}]u + \frac{\partial}{r\partial\psi} v_\psi(r,r_0,z,z_0,\psi,\psi_0)\} - \frac{i\omega p}{c^2(r,z,\psi)}$$
$$= \frac{i\delta(r-r_0)}{\omega r}\,\delta(z\text{-}z_0) \sum_{n=-\infty}^{\infty} \delta\,(\psi\text{-}\psi_0\text{-}2\pi n)\ . \qquad (1)$$

Here p and $\mathbf{v} = \{u, v_\psi, w\}$ are the fields of acoustic pressure, horizontal and vertical (w) components of the oscillatory velocity, c and ρ are the sound speed and density of the medium, ω is the cyclic frequency of radiation. Arguments r_0, z_0, ψ_0 are omitted below to abbreviate notation. Let values $z = \{h,\ H\}$ correspond to the location of the ocean bottom and ocean surface. We specify normalized value of density $\rho = 1$ in the water. The bottom can usually be considered stratified in layers, i. e., $\rho = \rho(z)$, although this is not a principal, since distortions from layered stratification can be taken into account if necessary, using the boundary condition of the general form:

$$p(r,H,\psi) = 0\ , \quad p(r,h,\psi) - \Omega(r,h,\psi)\, w(r,h,\psi) = 0\ , \qquad (2)$$

where, according to the definition, we introduced the function $\Omega(r,z,\psi) = p(r,z,\psi)/w(r,z,\psi)$ of acoustic impedance at level z. In horizontal plane (r,ψ) for functions p, v_ψ, w there are the condition of a 2π-periodicity, condition of the finiteness as $r \to (0,\infty)$ and continuity condition in passing through the interfaces between regular (layered) and irregular parts of a medium. If the sea medium is stratified in layered form and the impedance at the bottom exists in the form of fluid halfspace (or its part), the solution of problems (1)-(2) can be presented as a sum of propagating, leaky modes and field of a side wave (branch-cut term) [1, 2, 5]. If the medium includes horizontal heterogeneities $c(z,r,\psi)$ and $h(r,\psi)$, then, according to the method of transverse sections (the full two-way coupled-mode approach), the solution of problem (1)-(2) can be presented in the form of decomposition over local modes in each separate vertical section $r = \text{const}$ of the waveguide [1, 2] in the preset direction ψ:

$$p(r,\,z,\,\psi) = \sum_l \bar{\varphi}_{1\,l}\,(r,\,z,\,\psi)\; G_l(r,\,\psi)\ ,$$

$$w(r,\,z,\,\psi) = \sum_l \bar{\varphi}_{2\,l}\,(r,\,z,\,\psi)\; G_l(r,\,\psi)\ ,$$

$$u(r,\,z,\,\psi) = [i\omega\rho]^{-1} \sum_l \bar{\varphi}_{1\,l}\,(r,\,z,\,\psi)\; g_l(r,\,\psi)\ ,$$

$$v_\psi(r,\,z,\,\psi) = [i\omega\rho]^{-1} \sum_l \bar{\varphi}_{1\,l}\,(r,\,z,\,\psi)\; g_l^{\psi}(r,\,\psi)\ . \qquad (3)$$

Normalized eigenfunctions $\bar{\varphi}_{1l}(r,z,\psi)$, $\bar{\varphi}_{2l}(r,z,\psi) = [i\omega\rho(z)]^{-1}(\partial\bar{\varphi}_{1l}/\partial z)$ with a preset domain of definition $\mathcal{D}$ and local eigennumbers $\kappa_l(r,\psi)$ depend on r, ψ parametrically, and in each r-section they satisfy the layered eigenvalue problem. Substitution of modal presentations (3) into original Eqs.(1)-(2), taking into account orthonormalized eigenfunctions, leads to equations for the horizontal amplitudes $G_l(r,\psi)$, $g_l(r,\psi)$, $g_l^{\psi}(r,\psi)$ of decomposition (3) [6, 7]:

$$\frac{\partial}{\partial r}G_m(r,\psi) = g_m(r,\psi) - \sum_l G_l(r,\psi)\,V_{ml}(r,\psi) \quad , \tag{4a}$$

$$\frac{\partial}{r\partial\psi}G_m(r,\psi) = g_m^{\psi}(r,\psi) - r^{-1}\sum_l G_l(r,\psi)\,V_{ml}^{\psi}(r,\psi) \quad , \tag{4b}$$

$$\frac{\partial}{\partial r}g_m(r,\psi) + \frac{\partial}{r\partial\psi}g_m^{\psi}(r,\psi) = -\kappa_m^2(r,\psi)G_m(r,\psi) - \frac{1}{r}g_m(r,\psi)$$

$$-\sum_l [g_l(r,\psi)V_{ml}(r,\psi) + r^{-1}g_l^{\psi}(r,\psi)V_{ml}^{\psi}(r,\psi)]$$

$$-\overline{\varphi}_{1m}(z_0,r_0,\psi_0)\frac{\delta(r-r_0)}{2\pi r}\sum_{n=-\infty}^{\infty}\delta(\psi-\psi_0-2\pi n) \quad . \tag{4c}$$

Here $V_{ml}(r,\psi) = \int_{D}\frac{\overline{\varphi}_{1m}(z,r,\psi)}{\rho(z)}\frac{\partial\overline{\varphi}_{1l}(z,r,\psi)}{\partial r}dz$ and $V_{ml}^{\psi}(r,\psi) = \int_{D}\frac{\overline{\varphi}_{1m}(z,r,\psi)}{\rho(z)}\frac{\partial\overline{\varphi}_{1l}(z,r,\psi)}{\partial\psi}dz$ describe mode coupling caused by the horizontal variations of the medium.

3 Boundary problem reduction to the initial-value one

Let $\psi_0=0$ and simplify the problem assuming azimuth mode coupling is adiabatic, i. e., $V_{ml}^{\psi}(r,\psi) = 0$. Taking into account 2π-periodicity of functions, for ψ-harmonics of the field we obtain:

$$\{G_m(r,\psi), g_m(r,\psi)\} = \sum_{n=-\infty}^{\infty}\{G_m(r,n), g_m(r,n)\}exp(in\psi). \tag{5}$$

For preset ψ problem (4) can be reduced to the following (n is omitted, functions are scaled by the constant $1/2\pi r_0$):

$$\frac{\partial}{\partial r}G_m(r) = g_m(r) - \sum_l G_l(r)V_{ml}(r,\psi)\,, \tag{6a}$$

$$\frac{\partial}{\partial r}g_m(r) = [n^2r^{-2} - \kappa_m^2(r,\psi)]G_m(r) - \frac{1}{r}g_m(r) - \sum_l g_l(r)V_{ml}(r,\psi)$$

$$+\ 2a_m\delta(r - r_0) \quad . \tag{6b}$$

$a_m = -\overline{\varphi}_{1m}(z_0,r_0)/2$. Assume that irregular area with a point source occupies the part of a medium $r\in(L,L_0)$ (Fig.1). We place the source at the right interface $r_0\rightarrow L$, then boundary conditions to Eqs.(6) follow from the continuity of functions G_m and g_m , taking into account $g_m(r)$- jump at the source point:

$$g_m(L) + D_m(L)G_m(L) = -2a_m \quad , \tag{7a}$$

$$g_m(L_0) + d_m(L_0)G_m(L_0) = 0 \quad . \tag{7b}$$

Here $D_m(L) = \kappa_m^1 H_{n+1}^{(1)}(\kappa_m^1 L)/H_n^{(1)}(\kappa_m^1 L) - nL^{-1}$, $d_m(L_0) = \kappa_m^0 J_{n+1}(\kappa_m^0 L)/J_n(\kappa_m^0 L) - nL_0^{-1}$.

All the known approximate methods simplify this boundary problem replaced it by the initial-value problem. Usually, the backscattered field is ignored and irregular region is positioned to the far field of a source. However, the boundary problem (6)-(7) without any approximations can be reduced to the causal problem for imbedding equations [6-8]. For this aim we have to consider a parametric dependence of functions G_m, g_m on the position L of irregular region boundary L, where point source is located (or on that, if $r_0 > L$, normal modes incident from the source), i. e., $G_m(r) = G_m(r;L)$, $g_m(r) = g_m(r;L)$. Using standard procedure [9] of differentiating with respect to the parameter L and comparing the new and old problems we

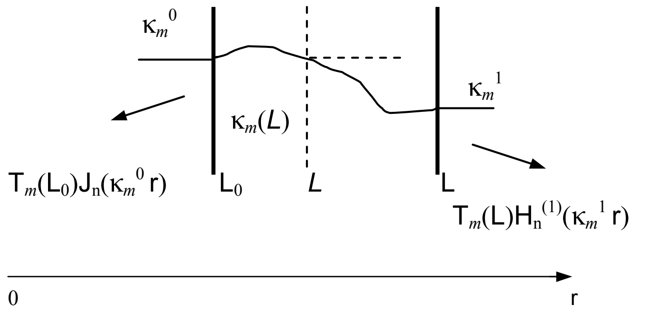

Fig.1 Point source is at the interface boundary L. $T_m(L_0)J_n(\kappa_m^0 r)$ and $T_m(L)H_n^{(1)}(\kappa_m^1 r)$ are outgoing fields of modes. For $|\kappa_m r|>>n$ asymptotics are valid for cylindrical functions Bessel J_n and Hankel $H_n^{(1)}$.

obtain for modes the system of coupled ordinary differential equations (ODE) instead of the boundary problem (6)-(7):

$$G_m(r;L) = G_m(r;r)exp\{\int_r^L d\xi\,[(\xi^{-1}-D_m(\xi)) - P_m(\xi)G_m(\xi;\xi) +$$

$$+ a_m^{-1}\sum_l V_{ml}(\xi)[G_l(\xi;\xi)(D_l(\xi) - D_m(\xi))/2 + a_l]]\} \quad , \tag{8a}$$

$$\frac{d}{dL}G_m(L;L) = [L^{-1}-2D_m(L)]G_m(L;L) - 2a_m - P_m(L)G_m^2(L;L)$$

$$+ a_m^{-1}\sum_{l\neq m}V_{ml}(L)[G_m(L;L)G_l(L;L)(D_l(L)-D_m(L))/2 + G_m(L;L)a_l$$

$$- G_l(L;L)a_m], \quad G_m(L;L)|_{L=L_0} = 2a_m/[d_m(L_0) - D_m(L_0)], \tag{8b}$$

$$P_m(\xi) = [D_m(\xi)/\xi - D_m^2(\xi) + (\partial/_{\partial\xi})D_m(\xi) + n^2/\xi^2 - \kappa_m^2(\xi)]/(2a_m).$$

Similar initial value equations are valid for functions $g_m(r;L)$. Deriving Eqs.(8) we imply that boundaries L,L_0 separated irregular area are consistent, as in reality, i. e., there are no jumps of the medium parameters for each r-sections, so $\kappa_m^1 = \kappa_m(L)$ and has to be differentiated with respect to L. Imbedding Eqs.(8) are already closed respectively each matrix function $G_m(r;L)$ and similar for $g_m(r;L)$. They satisfy the principle of dynamic causality, and well-developed methods to solve ODE numerically can be applied for them. Functions $G_m(L;L)$, satisfying Eqs.(8b), characterize backscattered field of modes at the irregular medium cross sections. Thus, in order to obtain the solution of boundary problem (4)-(7) without approximations and so, find sound field within the irregular medium, for preset ψ we have to solve Eqs.(8b), next calculate the integrals (8a), and sum up harmonics (5) and modes (3) for taken horizons of the source and observation. It is seen from Eqs.(8a), that backscattering within the medium influences both mode amplitude and phase. Below to illustrate our approach we consider in Cartesian coordinates (x,z) two-dimensional problem for the source of radiation $-\delta(x)\delta(z)$. Passage to this problem in Eqs.(6)-(8) corresponds to n = 0,1, $|\kappa_m r| >> 1$, or to such ψ-harmonics that satisfy the condition $|\kappa_m r| >> n$. In this case (8) can be simplified [9]:

$$G_m(x;L) = G_m(x;x)exp\{\int_x^L d\xi\,[i\kappa_m(\xi) - i\kappa_m'(\xi)G_m(\xi;\xi)/(2a_m)$$

$$+ a_m^{-1}\sum_l V_{ml}(\xi)[iG_l(\xi;\xi)(\kappa_m(\xi) - \kappa_l(\xi))/2 + a_l]]\}\,, \tag{9a}$$

$$\frac{d}{dL}R_m(L;L) = 2i\kappa_m(L)R_m(L;L) + \kappa_m'(L)[1 - R_m^2(L;L)]/(2\kappa_m)$$

$$+ a_m^{-1}\sum_{l\neq m}V_{ml}(L)\,a_l\,[(1-R_l)(1+R_m) - (\kappa_m(L)/\kappa_l(L))(1+R_l)$$

$$\times(1-R_m)]/2\,, \quad R_m(L_0;L_0) = 0\,. \tag{9b}$$

Here $\kappa_m'(x) = (\partial/\partial x)\kappa_m(x)$ and we write the problem solution via the backscattered field of modes $R_m(x;x) = i\kappa_m(x)G_m(x;x)/a_m - 1$, that satisfies Eqs.(9b).

4 Several examples for modeling in two dimensions

As a first example for simulation consider irregular waveguide with homogeneous water and sloping rigid bottom (RBM) for two low sound frequencies f = 8, 30 Hz, when there are small quantity of propagating modes within the waveguide. Let the source is located at the depth $z_0 - h = 0.5(H - h)$. Eqs.(8) have been simulated by the author's algorithms using the subroutines of ODE solving of the MATLAB product. In Figs.2,3 the charts of a field transmission loss in decibels (dB) are presented versus the distance x for several horizons of an observation. Level of

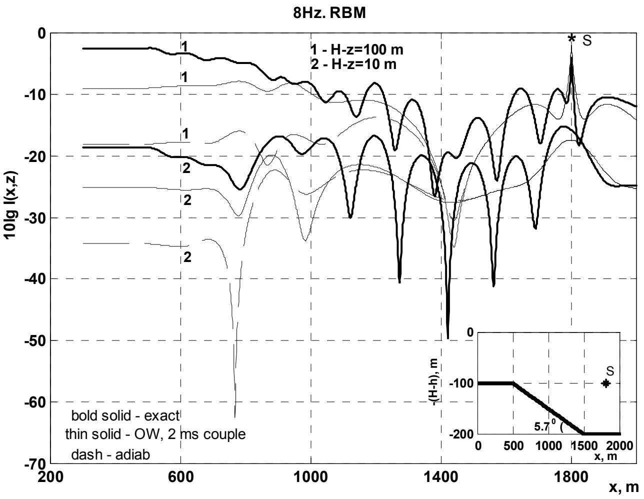

Fig.2 Transmission loss for indicated hydrology and two observation depths from the surface: 1 – 100 m, 2 – 10 m. Bold solid curves are the exact solution by Eqs.(9), thin curves are the one-way coupled-modes (OW), dashed curves are the adiabatic approximation. Point source S is at the depth 100 m and distant 300 m from the irregular area $x \in (L_0 = 500$ m, $L = 1500$ m), sound frequency is 8 Hz.

curves is scaled by the value $iH_0^{(1)}(k)/4$ ($k = \omega/c$ is the wave number in the water) of a free space pressure field respectively the distance $r = (x^2 + z^2)^{1/2} = 1$ m from radiating line tone source $-\delta(r)/r$. Sound field intensity behaviour is presented for the source that radiates within the layered part of the waveguide distant $d = |x_0 - L| = 300$ m from its irregular region. But at first, we calculate mode fields (9a) for the source placed at the boundary L, after that sound field can be expanded algebraically to the regular parts of a medium. In the considered example quantity of modes in the layered parts of the waveguide are varied in 2 times in passing through the irregular area. So, for 8 Hz there are one propagating mode $m = 1$ at $x < L_0$ and two modes $m = 1, 2$ at $x > L$. For 30 Hz there are 4 and 8 propagating modes correspondently. Passing through the irregular part of the waveguide propagating modes transform to evanescent ones and there is a strong backscattering nearby the sections (where $\kappa_m \approx 0$) of such transform. At that time, the other modes are considerably amplified due to coupling. As an example, for 8 Hz 2nd mode essentially amplifies 1st mode. In general these two modes form pressure field within the irregular part of waveguide for the distant source. However, if d decreases the influence of higher modes (evanescent ones) appears. Region of such influence $x \sim (\mathrm{Im}\,\kappa_m)^{-1}$, and occupies more and more portion of the irregular area if slope angle of the bottom increases. For d = 0 m (source is at the boundary L) even for the frequency 8 Hz we need take into account no less than 16 modes to describe the field nearby source better than 1% error. If frequency grows to 30 Hz, inside

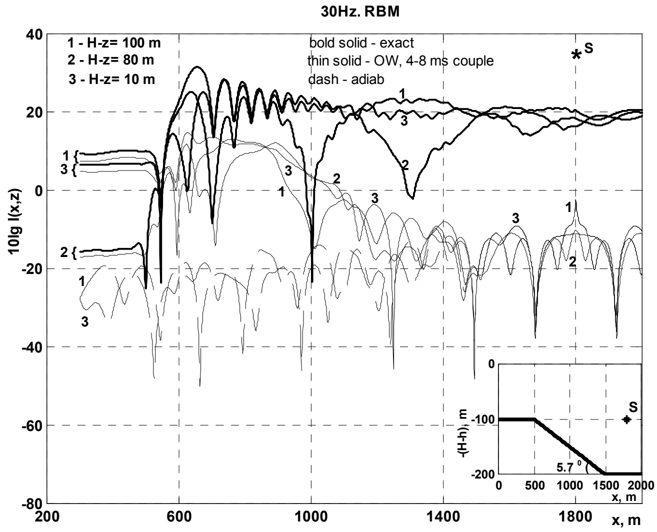

Fig.3 Similar to Fig.2 transmission loss for the frequency 30 Hz. Observation depths from the surface: 1 – 100 m, 2 – 80 m, 3 – 10 m.

the irregular area of the waveguide features of mode coupling such that odd modes 3, 5 and 7th are sharply amplified in series. As a result, for $x < L_0$ field is determined by the 3d propagating mode, and for $x > L$ 5th and 7th modes together with modes 1 and 2 contribute generally to entire field. The distant source case d = 300 m was specified here advisedly to compare with the results of one-way propagation approximate methods, since one-way approximation requires the source is placed far from the irregular area of a waveguide. It is known that adiabatic approximation ignores backscattering and mode coupling. Methods WKB and PE neglect backscattering and consider mode coupling to certain degree. In figures, adiabatic solution is indicated by dashed curves and is well seen due to considerably lower levels respectively the exact solution (9) (about 15-40 dB lower). Qualitative behaviour of adiabatic curves is also different from the exact ones. It is characterized by the gradual decrease along the all track of propagation and by the essential smoothness at frequency 8 Hz (in this case field is generally determined by the two first modes, and only by the 1st mode as $x < 530$ m). Whereas the exact solution due to backscattering and mode coupling is characterized by the essential level oscillations and its rise to the boundary $x = L_0$, that is more expressed for higher frequency 30 Hz. Thin solid curves in figures correspond to one-way coupled-modes (OW). In this case backscattering was neglected, so the transmission loss curves coincide with the adiabatic ones right side from the irregular area. At the same time mode coupling due to irregularities has been taken now into account (for all propagating modes, and this is already out of the scope of WKB). It is well seen that there is an intermediate sample between the exact and adiabatic solutions. Difference from the exact solution in the parts of waveguide now is varied. So, for 8 Hz it is about 8-10 dB right and left from the irregular area. For 30 Hz (Fig.3) the difference is minimum ≈ 3-5 dB as $x < L_0$, it is ≈ 10 dB within the irregular region and the highest ≈ 20-40 dB as $x > L$. Therefore as frequency grows OW approximation describes better the transmitted field ($x < L_0$), and worst of all (as adiabatic theory) it describes the field at $x > L$, where it is a combination of the direct and backscattered fields. Note also the fact that for higher frequency the difference between curves increases.

In reality, sea bottom is not perfectly rigid boundary, so above results can not be automatically applied to practice and require corrections. As the next model, consider irregular waveguide with the finite impedance of the bottom. That is the Pekeris waveguide problem (Fig.4). It is

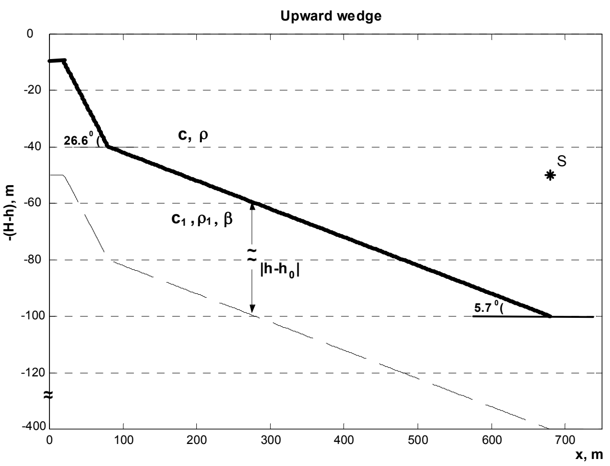

Fig.4 Irregular waveguide model with homogeneous water and upslope bottom $h(x)$ in the form of a halfspace, or a fluid sediment layer of the thickness $|h\text{-}h_0|$ = 300 m (dash line). There are the parameters: c = 1500 m/s, c_1 = 1650 m/s, $k_1 = \omega(1+i\beta)/c_1$ is the wave number in sediments, $m_\rho = \rho_1/\rho = 2$ is the ratio of densities, β = 0.005. Point source S location at the boundary L is shown by star.

well known that for the Pekeris waveguide model (PM) sound field can not be reduced only to the sum of discrete modes. Entire solution involves also the side wave field, that is the contribution of the continuous spectrum of values κ in the form of a branch-cut integral. Sediment halfspace exactly the reason of its appearing. Pekeris vertical branch cut (k_1, ∞) in the complex κ-plane was referenced for our modeling. As a result, sound field reduces to the sum of propagating and leaky modes and to the mentioned cut integral. For the irregular waveguide, if such integral has considered, calculations become very difficult. So the prospective way to avoid this complexity is considering the side wave as the contribution of the certain series of discrete eigenvalues κ_m, which approximate the continuous spectrum of κ. In papers [10, 11] one of the choices to make such approximation was proposed for layered waveguides. Fluid halfspace in the Pekeris model has been replaced by the finite thickness layer having the complex metric $|h\text{-}h_0|\exp(i\pi/4)$ and rigid lowest boundary. It was substantiated that the accuracy of such approximation (convergence to value of the Pekeris branch-cut integral) rapidly grows as $|h\text{-}h_0|^{-3}$, and slower, as $|h\text{-}h_0|^{-1}$, only if frequency is critical. We realized this way exactly to obtain the part of a solution due to the side wave in the irregular waveguide based on the Eqs.(6)-(9), taking thickness value $|h\text{-}h_0|$ = 300 m. Acoustical frequency for modeling has been taken f = 105 Hz, in order to the quantity of propagating modes in a waveguide was not large, but not less than 1. In the considering case of an upward wedge (Fig.4) quantity of modes decreases from 6 modes as distance $x > L$ = 680 m to 1 mode as $x < L_0$ = 20 m. In Figs.5, 6 below the field intensity loss are presented similar to Figs.2, 3. It is well seen that in this case of an upward wedge the backscattered field caused by the irregular area is large enough. This fact

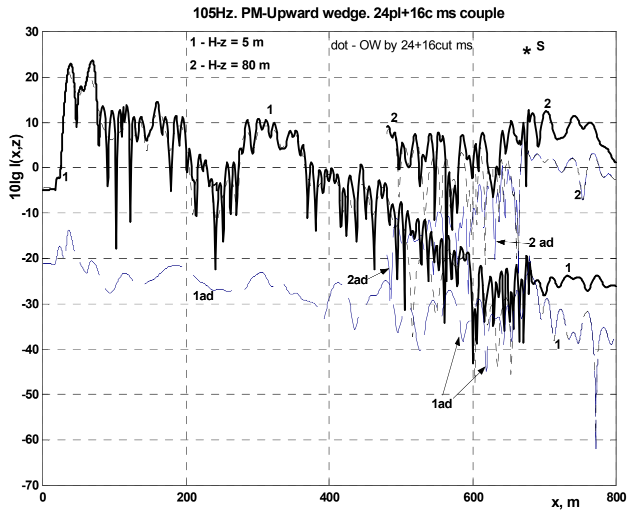

Fig.5 Transmission loss for hydrology in Fig.4 and two observation depths from the surface: 1 – 5 m, 2 – 80 m. Bold solid curves are the exact solution by Eqs.(9), dotted curves are one-way coupled-modes (OW), dashed curves are the adiabatic approximation. Point source S is located at the depth 50 m and at the interface L = 680 m between the layered and irregular areas, sound frequency is 105 Hz.

results in the considerable divergence of curves corresponding to the exact solution and to one-way coupled-mode theory, that is similar to previous RBM-case. Note that OW-dependences have been obtained based on the field exact representation (9a) if backscattering was neglected. For this aim in Eq.(9a) we have to substitute $G_m(x;x) = a_m/[i\kappa_m(x)]$, since $R_m(x;x) = 0$. Then, we obtain

$$G_m(x;L) = a_m[i\kappa_m(x)\kappa_m(L)]^{-1/2} \exp\Big\{\int_x^L d\xi\,\Big[i\kappa_m(\xi) + a_m^{-1}\sum_l V_{ml}(\xi)a_l\,[(\kappa_m(\xi)-\kappa_l(\xi))/2\kappa_l(\xi)+1]\Big]\Big\}\,, \quad (10)$$

In adiabatic case besides must be $V_{ml}(r) = 0$. From Eq.(9b) it follows that backscattering and mode coupling effects are closely related, so that strong backscattering can be observed if strong coupling of modes is present in the waveguide. For the case considered (Fig.5) maximum value of the backscattering is observed at the entry to wedge, where point source is located. As far as modes penetrate inside the wedge the coupling between decreasing quantity of essential modes become very important. That is why OW-dependences correspond better and better to the exact curves whereas adiabatic curves (they lie noticeably lower) on the contrary correspond worse and worse. Mode coupling leads to general increase of a field level in the wedge, but at its way out, $x < L_0$, level rapidly decays, since for these distances, $x < 20$-30 m, sound field is determined only by the 1st mode, though it is amplified respectively the adiabatic approximation. Side wave presence is well seen even for the depth 5 m, especially at the wedge entry, $x \approx L = 680$ m. In this region the side wave contribution varies the intensity of field in 5-15 dB.

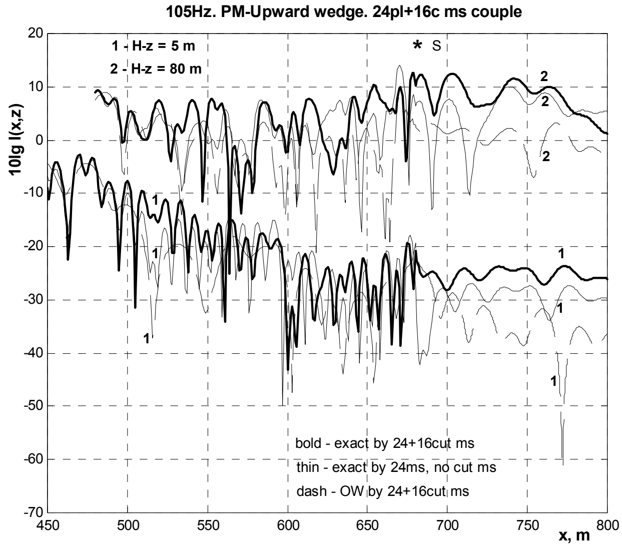

Fig.6 Transmission loss similar to Fig.5. Bold solid curves are the exact solution, thin solid curves are the solution without side wave and dashed curves are one-way coupled-modes.

5 Conclusion

In this paper the new prospective approach to the low-frequency acoustic field modeling in the irregular ocean waveguides is presented. It is based on the boundary problem statement for acoustic wave equations describing the modes in horizontal plane and its reduction to the equivalent initial-value problem involving coupled matrix ordinary differential equations. It allows in many cases obtaining the useful form of the approximations (e. g., writing in the explicit integral form the solution to one-way coupled-modes approximation and to the particular cases of the backscattered field of modes) as well as the numerical analysis of sound propagation in 2D and 3D waveguides by well-developed methods of ODE solving. For the low frequencies of sound and 2D waveguide model with a slope rigid bottom the considerable difference is obtained between the transmission loss due to the exact solution (9) and the loss corresponding to the approximate methods. However, this fact is of no surprise since for the considered situation the backscattered field being the part of entire field is large enough due to both the rigid bottom and the non-smooth its profile. The question is how much the obtained results will change in the case of more realistic bottom model? Already by RBM-example we have answered that for the non rigid sea bed one can wait the similar results for higher frequency of sound (~ 100 Hz), since as it was established the backscattering increases if frequency grows (if the quantity of propagating coupled modes rises that appear in decrease of difference κ_m - κ_l). Indeed, by the example of the irregular Pekeris waveguide it was shown that strong effects of the backscattering and mode coupling can be observed also in the case of a fluid absorbing sediments forming the penetrable bottom. Due to this fact, one-way coupled-modes often can not properly describe sound field in various parts of a waveguide, even more so for adiabatic theory. Side wave as the branch-cut contribution to entire field taken into account does not change the quality features of transmission loss, if at least 1 propagating mode is presented within the irregular waveguide. Nevertheless, this contribution creates the essential quantity corrections for loss in the waveguide. These corrections reach 5-15 dB depending on the distance and horizons of the source and the observation.

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