The directivity of the forced radiation of sound from panels and openings including the shadow zone

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This paper presents a method for calculating the directivity of the radiation of sound from a two dimensional panel or opening, whose vibration is forced by the incidence of sound from the other side. The directivity of the radiation depends on the angular distribution of the incident sound energy. For panels or openings in the wall of a room, the angular distribution of the incident sound energy is predicted using a physical model which depends on the sound absorption coefficient of the room surfaces. For an opening at the end of a duct, the sound absorption coefficient model is used in conjunction with a model for the directivity of the sound source in the duct. For angles of radiation approaching 90 degrees to the normal to the panel or opening, the effect of the diffraction by the panel or opening, or by the finite baffle in which the panel or opening is mounted, is included. A simple empirical model has been developed to predict the diffraction of sound into the shadow zone when the angle of radiation is greater than 90 degrees to the normal to the panel or opening. The method is compared with published experimental results.

1 Introduction

This paper describes a theoretical method for predicting the directivity of the sound radiated from a panel or opening excited by sound incident on the other side. This directivity needs to be known when predicting the sound level at a particular position, such as that due to the sound radiation from a factory roof, wall, ventilating duct or chimney flue. There is surprisingly little information on how to predict this directivity in the scientific literature. Most of this information is based on limited experimental data or its basis cannot be determined.

2 Theory

The effective impedance $Z_e(\phi)$ of a finite panel in an infinite baffle to a plane sound wave incident at an angle of ϕ to the normal to the panel is

$$Z_e(\phi) = Z_{wfi}(\phi) + Z_{wft}(\phi) + Z_{wp}(\phi) \tag{1}$$

where

$Z_{wfi}(\phi)$ is the wave impedance of the fluid as experienced by the finite panel in an infinite baffle, whose vibration is due to a plane sound wave incident at an angle of ϕ to the normal to the panel, on the side from which the plane sound wave is incident (this is the fluid loading on the incident side),

$Z_{wft}(\phi)$ is the wave impedance of the fluid as experienced by the finite panel in an infinite baffle, whose vibration is due to a plane sound wave incident at an angle of ϕ to the normal to the panel, on the side opposite to which the sound is incident (this is the fluid loading on the non-incident or transmitted side) and

$Z_{wp}(\phi)$ is the wave impedance of the finite panel in an infinite baffle to a plane sound wave incident at an angle of ϕ to the normal to the panel, ignoring fluid loading.

It will be assumed that the fluid wave impedances on both sides are the same and the imaginary part of the fluid wave impedance will be ignored. That is

$$Z_{wfi}(\phi) = Z_{wft}(\phi) = \rho c \sigma(\phi) \tag{2}$$

where ρ is the density of the fluid, c is the speed of sound in the fluid and $\sigma(\phi)$ is the radiation efficiency into the fluid of one side of the finite panel in an infinite baffle, whose vibration is due to a plane sound wave incident at an angle of ϕ to the normal to the panel.

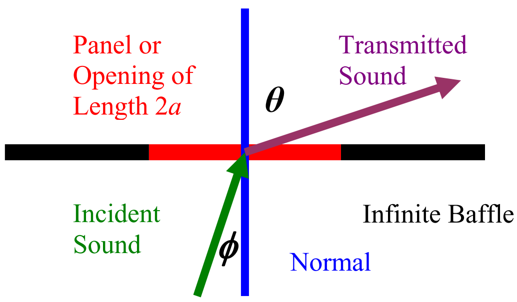

Fig.1 Sound incident at an angle of ϕ to the normal to a panel or opening and radiated at an angle of θ to the normal.

Reflections at the panel edges are ignored. The rms normal velocity $v_{rms}(\phi)$ of the panel due to a plane sound wave incident at an angle of ϕ to the normal to the panel which exerts an rms pressure $p_{irms}(\phi)$ is

$$v_{rms}(\phi) = \frac{p_{irms}(\phi)}{2\rho c \sigma(\phi) + Z_{wp}(\phi)}. \tag{3}$$

The transmitted rms sound pressure $p_{trms}(\theta,\phi)$ which is radiated by the panel on the non-incident side to a receiving point which is at an angle of θ to the normal to the centre of the panel and a large distance from the panel (see Fig.1) is [1]

$$p_{trms}(\theta,\phi) \propto v_{rms}(\phi) \frac{\sin\left[ka(\sin\theta - \sin\phi)\right]}{ka(\sin\theta - \sin\phi)} \tag{4}$$

where k is the wave number of the sound and $2a$ is the average length across the panel or opening in the plane containing the receiver and the normal to the panel or opening. Thus

$$p_{trms}(\theta,\phi) \propto \frac{p_{irms}(\phi)}{2\rho c \sigma(\phi) + Z_{wp}(\phi)} \frac{\sin\left[ka(\sin\theta - \sin\phi)\right]}{ka(\sin\theta - \sin\phi)}. \tag{5}$$

The case where the incident sound is generated by a sound source in a room or duct is now considered. We assume that the sound pressure waves are incident at different angles ϕ with random phases and mean squared sound pressures which are proportional to a weighting function $w(\phi)$.

$$|p_{irms}(\phi)|^2 \propto w(\phi). \tag{6}$$

The weighting function is to account for the fact that sound waves at grazing angles of incidence will have had to suffer more wall collisions and therefore be more attenuated before reaching the panel. The total mean square sound pressure $|p_{Trms}(\theta)|^2$ at the receiving point is

$$|p_{Trms}(\theta)|^2 \propto \int_{-\pi/2}^{\pi/2} \frac{w(\phi)}{|2\rho c\sigma(\phi)+Z_{wp}(\phi)|^2}\left\{\frac{\sin\left[ka(\sin\theta-\sin\phi)\right]}{ka(\sin\theta-\sin\phi)}\right\}^2 d\phi \,. \tag{7}$$

The case when sound is incident from a source in a free field at an angle θ to the normal to the panel and the panel radiates at all angles ϕ into a room or duct is also of interest. In this case the weighting function $w(\phi)$ is to account for the fact that sound waves radiated at grazing angles will have had more wall collisions and therefore be more attenuated before reaching the receiving position which is assumed to be a reasonable distance from the panel or opening which is radiating the sound. In this second case, we have to integrate over all angles of radiation ϕ because of the reverberant nature of the sound. For this case, the impedance terms in the integral are functions of θ rather than ϕ and can be taken outside the integral. However in this study both cases are calculated using the formula for the first case which is shown above. This is because both cases should be the same by the principle of reciprocity and it is not clear which form of the formula is more appropriate.

For large values of ka, the two cases of the formula will be similar. If ka is much greater than 1, the function

$$\left\{\frac{\sin\left[ka(\sin\theta-\sin\phi)\right]}{ka(\sin\theta-\sin\phi)}\right\}^2 \tag{8}$$

has a sharp maximum at $\phi=\theta$ and is symmetrical in both ϕ and θ about the point $\phi=\theta$. We can exploit these facts by evaluating the impedance terms for the first case at $\phi=\theta$ and taking them out side the integral. This gives the formula for the second case.

To derive the angular weighting function, we assume that the sound source is distance b from the surface of the room containing the panel or opening and that the room width is g in the plane containing the incident sound ray (see Fig.2). If the sound ray is incident at an angle of ϕ to the normal to the panel or opening, it travels a minimum distance of $b\tan\phi$ parallel to the wall containing the panel or opening before hitting the wall. The sound which travels this minimum distance hits the wall approximately

$$n=\frac{b}{g}\tan\phi \tag{9}$$

times before reaching the panel or opening, where n is allowed to be a real number rather than an integer in order to give a smooth weighting function. If the sound absorption coefficient of the walls of the room is α, the sound intensity incident at an angle of ϕ to the normal is proportional to

$$w(\phi)=(1-\alpha)^n=(1-\alpha)^{\frac{b}{g}\tan|\phi|}. \tag{10}$$

Equation (10) gives us the weighting function $w(\phi)$. If α is zero, a uniform diffuse field will be obtained. For rigid walled ducts, a value of α equals 0.05 is recommended.

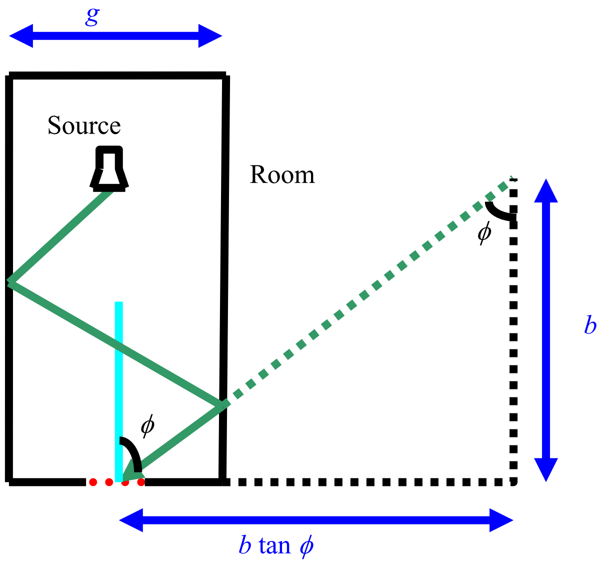

Fig.2 Calculating the number of wall reflections before sound hits the panel or opening at an angle of ϕ to the normal.

In this study we use the radiation efficiency of a panel or opening of length $2a$ and width $2d$, which we approximate with the following equation [1].

$$\sigma(\phi)=\begin{cases}\dfrac{1}{\dfrac{\pi}{2k^2ad}+\cos\phi} & \text{if } |\phi|\le\phi_l\\[2ex] \dfrac{1}{\dfrac{\pi}{2k^2ad}+\dfrac{3\cos\phi_l-\cos\phi}{2}} & \text{if } \phi_l<|\phi|\le\dfrac{\pi}{2}\end{cases} \tag{11}$$

where

$$\phi_l=\begin{cases}0 & \text{if } ka\le\dfrac{\pi}{2}\\ \arccos\left(\sqrt{\dfrac{\pi}{2ka}}\right) & \text{if } ka>\dfrac{\pi}{2}\end{cases} \tag{12}$$

and k is the wave number of the sound and $2a$ is the length of the panel in the direction of the receiver.

For an opening with no panel in an infinite baffle we put $Z_{wp}(\phi)=0$. For a finite panel in an infinite baffle we use the infinite panel result for $Z_{wp}(\phi)$. This result is expected to be the correct result when averaged over frequency, because this approach gives the correct result for point impedances when averaged over frequency and position on a finite panel.

$$Z_{wp}(\phi)=m\omega\left\{j\left[1-\left(\frac{\omega}{\omega_c}\right)^2\sin^4(\phi)\right]+\eta\left(\frac{\omega}{\omega_c}\right)^2\sin^4(\phi)\right\} \tag{13}$$

where m is the surface density (mass per unit area) of the panel, η is the damping loss factor of the panel, ω_c is the angular critical frequency of the panel and ω is the angular frequency of the sound.

In a duct, the directivity of the sound source is also included. The sound source is modeled as a line source of length $2r$ where r is the radius of the sound source. The directivity of the sound source is proportional to

$$\left[\frac{\sin(kr\sin\phi)}{kr\sin\phi}\right]^2 \tag{14}$$

where k is the wave number. The value of $2r$, which gives best agreement between experimental results and the theory of this paper varies over a wide range. However the average value of $2r$ over a large number of experimental results is approximately the wavelength λ of sound in air. This makes kr equal to π.

For angles of radiation close to 90° to the normal to the panel or opening, the effect of the diffraction by the panel or opening or by the finite baffle in which the panel or opening is mounted needs to be included [2]. $p(\theta)$ is the ratio of the increased sound pressure to the sound pressure without the baffle for an angle of incidence or radiation of θ. The baffle is of length $2L$ in the plane containing the receiver (or source) and the normal to the baffle and of width $2W$ in the direction at right angles to the above mentioned plane. Note that in [2], the length and width of the baffle were assumed to be equal. The increase in sound pressure due to radiation (or incidence) of sound pressure of wave number k normally from (or onto) a panel or opening in a baffle is

$$p(0) = 1 + p_W p_L \tag{15}$$

where

$$p_W = \begin{cases} \sin(kW) & \text{if } kW \le \dfrac{\pi}{2} \\ 1 & \text{if } kW > \dfrac{\pi}{2} \end{cases} \tag{16}$$

and

$$p_L = \begin{cases} \sin(kL) & \text{if } kL \le \dfrac{\pi}{2} \\ 1 & \text{if } kL > \dfrac{\pi}{2} \end{cases}. \tag{17}$$

The limiting angle below which the sound pressure does not vary with angle of radiation (or incidence) is θ_m. Notice that if $L = a$, $\theta_m = \phi_l$.

$$\theta_m = \begin{cases} 0 & \text{if } kL \le \dfrac{\pi}{2} \\ \arccos\left(\sqrt{\dfrac{\pi}{2kL}}\right) & \text{if } kL > \dfrac{\pi}{2} \end{cases}. \tag{18}$$

There is no increase of sound pressure at grazing angles of transmission (or incidence).

$$p\left(\frac{\pi}{2}\right) = 1. \tag{19}$$

$p(\theta)$ is obtained by linear interpolation in $\cos(\theta)$. Note that this is different from [2], where the linear interpolation was carried out in θ.

$$p(\theta) = \begin{cases} p(0) \\ \text{if} \cos(\theta) \ge \cos(\theta_m) \\ \dfrac{p(0)\cos(\theta) + p\left(\dfrac{\pi}{2}\right)\left(\cos(\theta_m) - \cos(\theta)\right)}{\cos(\theta_m)} \\ \text{if } \cos(\theta_m) > \cos(\theta) \ge 0 \end{cases}. \tag{20}$$

The relative sound pressure level $L(\theta)$ in the direction θ is

$$L(\theta) = 10\log_{10}\left(\left|p_{Trms}(\theta)\right|^2 p^2(\theta)\right) - 20\log_{10}\left(\left|p_{Trms}(0)\right|^2 p^2(0)\right). \tag{21}$$

If the transmission is into the shadow zone, that is $\frac{\pi}{2} < |\theta| \le \pi$, then the above calculations are carried out for $\theta = \frac{\pi}{2}$ and the product $\left|p_{Trms}\left(\frac{\pi}{2}\right)\right|^2 p^2\left(\frac{\pi}{2}\right)$ in equation (21) is multiplied by the following diffraction correction.

$$D(\theta) = \frac{1}{1 - kz\cos(\theta)} \tag{22}$$

where

$$z = \frac{1}{\dfrac{1}{L} + \dfrac{1}{W}}. \tag{23}$$

In practical situations, scattering from turbulence and other objects will place a lower limit on the relative sound pressure level. Let $L_{\max}$ be the maximum value of $L(\theta)$. It is assumed that the scattered sound level is L_S dB below $L_{\max}$. The predicted observed relative sound pressure level $L_O(\theta)$ is

$$L_O(\theta) = 10\log_{10}\left(10^{L(\theta)/10} + 10^{(L_{\max} - L_S)/10}\right). \tag{24}$$

L_S would usually be expected to be greater than 20 dB. To predict the shadow zone data in [3], a value of L_S equals 22 dB was used.

3 Comparison with experiment

The theory developed in this paper is primarily for a rectangular opening, panel or baffle. For a circular opening, panel or baffle, what value should be used for the length and width? In this paper, the length and width are set equal to the diameter. Other possibilities are $\frac{\pi}{4}$ or $\frac{\sqrt{\pi}}{2}$ times the diameter. The theoretical results for ducts given in this section are computed using a wall absorption coefficient of 0.05 and a duct source directivity equal to that of a line source of length equal to the wavelength of the sound.

Croft [4] makes measurements on a duct of diameter 0.112 m and length 0.75 m in an anechoic room. He uses third octave bands of random noise from 6.3 to 20 kHz and measures the sound pressure level relative to 0° at angles of 45°, 60°, and 90°. The average difference between Croft's measurements and the theory presented in this paper is -0.1 dB.

Sutton [5] measures in an anechoic room with third octave bands of noise at frequencies of 0.8, 1.25, 2, 3.15, 4, 6.3, 10, 16, 25, and 40 kHz. He measures the sound pressure level relative to 0° at angles of 15°, 30°, 45°, 60°, 90°, 120°, 150°, and 180°. For a circular duct of diameter 0.085 m with a length of 0.75 m in a baffle of diameter 0.135 m, the average differences between his two separate measurements and the theory presented in this paper are 0.2 and -0.2 dB. A measurement using pure tones gives an average difference of -2.8 dB. A square duct of 0.08 by 0.08 m with length 0.75 m in a baffle of 0.13 by 0.13 m gives an average difference of -0.7 dB.

Sutton also makes measurements on a rectangular duct measuring 0.08 by 0.04 m with length 0.075 m in a baffle measuring 0.13 by 0.09 m. With the 0.08 m dimension in the plane of measurement, the average difference between experiment and theory is -1.0 dB. With the 0.04 m dimension in the plane of measurement, the average difference between experiment and theory is -1.2 dB. He also uses a rectangular duct measuring 0.12 by 0.04 m with length 0.075 m in a baffle measuring 0.17 by 0.09 m. With the 0.12 m dimension in the plane of measurement, the average difference between experiment and theory is 0.6 dB. With the 0.04 m dimension in the plane of measurement, the average difference between experiment and theory is -0.8 dB.

Dewhirst [6] makes measurements in an anechoic room with third octave bands of noise from 100 Hz to 16 kHz. He measures the sound pressure level relative to 0° at angles in steps of 10° from 10° to 180°. For a square duct measuring 0.12 by 0.12 m of length 0.75 m in a baffle of 0.268 by 0.268 m, the average difference between his results and the theory of this paper is -1.4 dB. He also makes measurements on a rectangular duct measuring 0.16 by 0.08 m with length 0.075 m in a baffle measuring 0.184 by 0.104 m. With the 0.16 m dimension in the plane of measurement, the average difference between experiment and theory is -0.1 dB. With the 0.08 m dimension in the plane of measurement, the average difference between experiment and theory is -1.1 dB. He also uses a rectangular duct measuring 0.24 by 0.08 m with length 0.075 m in a baffle measuring 0.264 by 0.104 m. With the 0.24 m dimension in the plane of measurement, the average difference between experiment and theory is 0.2 dB. With the 0.08 m dimension in the plane of measurement, the average difference between experiment and theory is -1.6 dB. Dewhirst also makes measurements on a square duct lined with sound absorbing material, but the results do not appear to make sense. This is probably due to the high attenuation in the duct because of the sound absorbing material and the difficulty of reducing breakout noise from the loudspeaker enclosure.

Li [7] measures in an anechoic room with third octave bands of noise at frequencies from 0.5 to 25 kHz. He measures the sound pressure level relative to 0° in steps of 1° at angles from 1° to 180°. For a circular duct of diameter 0.13 m with a length of 0.75 m in a baffle of diameter 0.26 m, the average difference between his measurements and the theory presented in this paper is -2.4 dB. Without the baffle, the average difference is -1.7 dB. He also makes measurements on a rectangular duct measuring 0.16 by 0.08 m with length 0.075 m in a baffle measuring 0.352 by 0.272 m. With the 0.16 m dimension in the plane of measurement, the average difference between experiment and theory is 0.8 dB. With the 0.08 m dimension in the plane of measurement, the average difference between experiment and theory is -1.3 dB. The average of the 18 average differences between theory and experiment for measurements on ducts in an anechoic room is -0.6 dB.

Neish [8] makes outdoor measurements over the plane of the ground on horizontal circular ducts with diameters of 0.4 and 1.22 m. He presents his results in octave bands from 31.5 Hz to 8 kHz and measures the sound pressure level relative to 0° in steps of 15° at angles from 15° to 165°. Potente, Gauld and Day [9] make outdoor measurements over the plane of the ground. They measure the sound pressure level relative to 0° in third octave bands from 50 Hz to 10 kHz in steps of 15° at angles from 15° to 165°, although their paper presents their results in octave bands. They measure 0.305, 0.61 and 0.914 m diameter ducts of different lengths at different distances from the duct mouth.

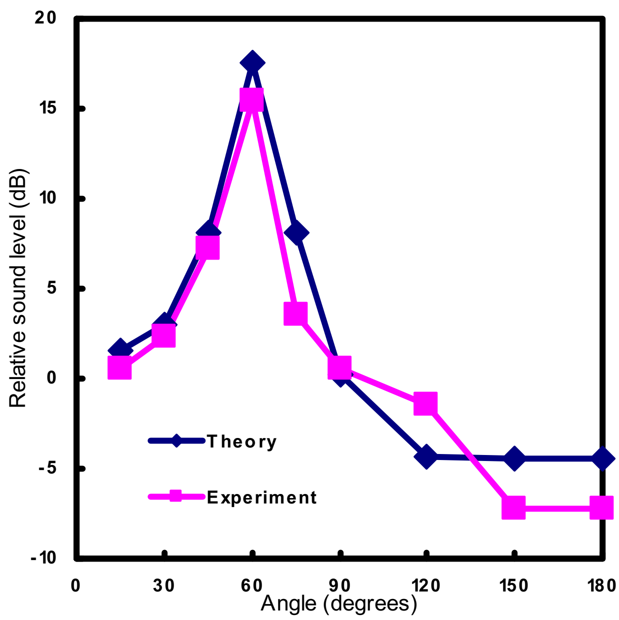

Fig.3 Comparison of the experimental and the theoretical sound pressure level relative to 0° of a 6 mm glass window at 3.15 kHz as a function of angle of incidence.

The average difference between experiment and theory for all the above outdoor measurements is 2.6 dB. This difference is thought to be due to the breakout of noise via the duct walls and the scattering of sound from atmospheric turbulence and objects, including the ground, in the vicinity of the duct. Apart from being of larger cross sectional area than the ducts used in the anechoic room measurement, the ducts used for the above outdoor measurements have less treatment to reduce breakout noise. The differences between experiment and usually increase when the duct length and measurement distance increase. This is consistent with this difference being due to breakout noise from the duct walls. Another difference is that the measurement distances for the anechoic room measurements are about 5 times the length of the ducts, while the measurement distances for the above outdoor measurements are less than or equal to the duct lengths.

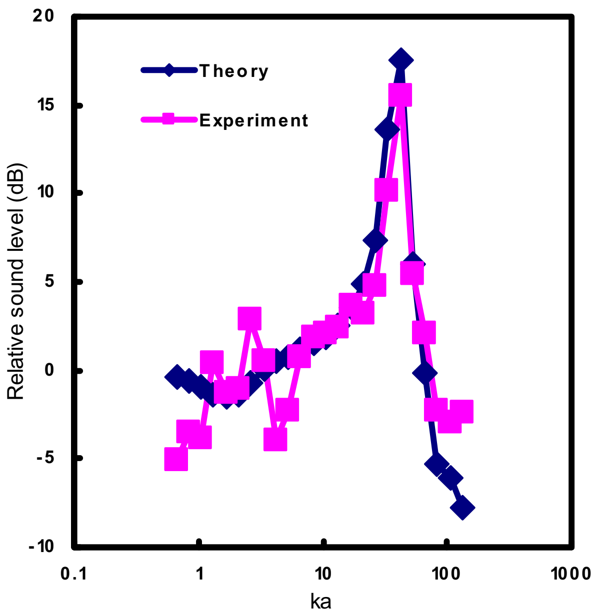

Fig.4 Comparison of the experimental and the theoretical sound pressure level relative to 0° of a 6 mm glass window at 60° as a function of ka .

Stead [3] measures the sound insulation directivity of a window installed in one wall of a room. The sound is incident at an angle to the normal to the window from outside the room. This is the opposite direction to the calculation method used in this paper, but is expected to give similar results because of the principle of reciprocity. The window is 1.45 m wide by 2.12 m high. The glass is 6 mm thick. The wall of the room containing the window is part of the external wall of a larger building which serves as a baffle. The internal dimensions of the room are 2.88 m wide by 3 m high by 5.12 m deep. The loudspeaker is 20 m from the middle of the window. The edge of the building in the direction of the measurements is 11 m from the centre of the window. Thus the baffle length is set to twice this distance, namely 22 m. Stead's measured reverberation times are used to calculate the average wall absorption coefficients of the room for use in the calculation of the weighting function. As noted earlier, the scattered sound level is set to 22 dB below the coincidence peak. The surface density of the glass is 14.4 kg/m^2, the critical frequency is 2250 Hz and the in situ damping loss factor is 0.1. The average difference between experiment and theory is 0.1 dB.

Fig.3 compares Stead's measurements and the theoretical sound pressure level relative to 0° at 3.15 kHz as a function of angle of incidence. Fig.4 compares Stead's measurements and the theoretical sound pressure level relative to 0° at an angle of incidence of 60° as a function of frequency expressed in terms of the product ka where k is the wave number and $2a$ is the length of the window in the horizontal plane of measurement. Both these figures show the coincidence peak.

4 Conclusion

The theoretical model presented in this paper can be used to predict the sound pressure level radiated at a particular angle to the normal of a panel or opening, relative to the sound pressure level radiated in the direction of the normal.

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