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A solution to the problem of simultaneous classification and localization of underwater objects from their acoustic field

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An approach to the solutions to the problem of simultaneous classification and localization of underwater objects from its acoustical field is presented. It is shown that both problems (classification and localization) have some common features that allow to consider them as one mutual problem which can be solved by maximum likelihood method.

1 Introduction

This paper presents one of the solutions to the problem of simultaneous classification and localization of underwater objects from their acoustic field. The idea is based on the following prerequisites:

a) The individual problems, i.e., the classification and localization ones, rely on the common physical basis and, consequently, are solved with the use of the same parameters of the signal received from the detected object. It is important to know that these parameters as a rule are mutually dependent and their statistical properties greatly depend on the current underwater observation conditions.
b) The class of the object is one of its parameters as well as the object coordinates. The difference is that the class can take only discrete values from a known limited denumerable set (the alphabet of classes).
c) It is known that, by taking into account the class of the object, one can improve the accuracy of the object localization, and, vice versa, knowing the coordinates of the object facilitates its classification. Thus, the simultaneous solution of the two aforementioned problems should improve the efficiency of solving either of them.
d) The simultaneous solution of the two problems should allow one to significantly simplify the total computational algorithm at the expense of the elimination of similar algorithms used for solving these problems separately.

The problems of classification and localization of underwater objects were considered independently by a number of authors [1,2].

2 The sense of the approach

The combined problem considered below is mathematically formulated as a problem of obtaining the maximum likelihood estimates (MLE) of the class of the target ω and the vector of coordinates $\mathbf{U}$ from the vector of the estimated signal parameters $\hat{\mathbf{X}}$. According to the theory of optimal estimation [3], the solution to the formulated problem has the form:

$$(\hat{\omega},\hat{\mathbf{U}}) = arg \max_{\omega,\mathbf{U}} g_{\hat{\mathbf{X}}/\omega,\mathbf{U}}(\hat{\mathbf{X}}) \quad , \tag{1}$$

where $g_{\hat{\mathbf{X}}/\omega,\mathbf{U}}(\mathbf{x})$ is the conditional probability density function (PDF) of the vector $\hat{\mathbf{X}}$ depending on the discrete target class $\omega=\omega_j,\ j=1,...,m$ and the utter coordinate vector $\mathbf{U}$.

Hence, to determine the maximum likelihood estimates of the target class ω and the coordinate vector $\mathbf{U}$, it is sufficient to obtain an expression for the conditional PDF of the vector $\hat{\mathbf{X}}$, substitute the measured value of the vector $\hat{\mathbf{X}}$ into it, and determine the maximum of the resulting likelihood function with respect to ω and $\mathbf{U}$. Then, the values ω and $\mathbf{U}$, at which the maximum is obtained, will be the maximum likelihood estimates.

According to [4], if the normalization conditions are met, the function $g_{\hat{\mathbf{X}}/\omega,\mathbf{U}}(\hat{\mathbf{X}})$ can be considered as the combined PDF of the scalar ω and the vector $\mathbf{U}$. On the basis of this function, the estimates of the efficiency of the accepted solution can be obtained, namely, the *a posteriori* probability of the correctness of the realized classification

$$P(\hat{\omega}) = \frac{g_{\hat{\mathbf{X}}/\omega,\hat{\mathbf{U}}}(\hat{\mathbf{X}})}{\sum_{j=1}^{m} g_{\hat{\mathbf{X}}/\omega_j,\hat{\mathbf{U}}}(\hat{\mathbf{X}})} \tag{2}$$

and the correlation matrix of the estimated coordinate with the *kl*th element calculated by the formula

$$\left\|K(\hat{\mathbf{U}})\right\|_{kl} = \int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}(U_k-\hat{U}_k)(U_l-\hat{U}_l)g_{\hat{\mathbf{X}}/\hat{\omega},\mathbf{U}}(\hat{\mathbf{X}})d\mathbf{U} \tag{3}$$

It follows from the above that, in the simultaneous classification and determination of coordinates of underwater objects, the main procedure is the calculation of the conditional PDF $g_{\hat{\mathbf{X}}/\omega,\mathbf{U}}(\mathbf{x})$, or, more precisely, $g_{\hat{\mathbf{X}}/\omega,\mathbf{U},\hat{\mathbf{\Gamma}}^{(1)}}(\mathbf{x})$, because, it is known that this PDF strongly depends on the current observation conditions characterized by the vector $\hat{\mathbf{\Gamma}}^{(1)}$ of the parameters which can be estimated.

Let us consider the procedure for computing the conditional PDF $g_{\hat{\mathbf{X}}/\omega,\mathbf{U},\hat{\mathbf{\Gamma}}^{(1)}}(\mathbf{x})$, which is pivotal in the proposed algorithm. A stochastic model is basic to the computation of the sought PDF's [5]. It relates the feature estimate vector $\hat{\mathbf{X}}$ to the object state vector $\mathbf{\Lambda}$ (which includes the object's noise radiation (noise reflection), position and motion parameters), to the observation condition parameter vector $\mathbf{\Gamma}$, and to the feature measurement error vector $\mathbf{Y}$. The vector $\mathbf{\Gamma}$ includes both on-line measurable parameters lumped into the vector $\mathbf{\Gamma}^{(1)}$ (such as vertical distribution of sound velocity and signal-to-noise ratio), and parameters lumped into the vector $\mathbf{\Gamma}^{(2)}$ that present certain difficulties for on-line measurements (such as the bulk reflection coefficients and the coefficients of reflection from the waveguide walls). In reduced form, the stochastic model is written as

$$\hat{\mathbf{X}} = \varphi(\mathbf{U}, \mathbf{\Lambda}, \mathbf{\Gamma}^{(1)}, \mathbf{\Gamma}^{(2)}, \mathbf{Y}) \tag{4}$$

The vector function $\varphi(.)$ covers the fundamental physical relationships involved in the radiation, propagation, and reception of signals and the measurement of their parameters in the presence of noise, and may therefore be regarded as a nonrandom function of random arguments. Strictly speaking, this function $\varphi(.)$ is random with random arguments. However, to simplify the mathematics, the function $\varphi(.)$ can be considered as a nonrandom one with its random behavior being attributed to its random arguments.

The stochastic model permits classification features to be judged as dependent or independent. Features are independent if the arguments of their stochastic models are independent, and vice versa.

Using stochastic model Eq.(4) the conditional PDF $g_{\hat{\mathbf{X}}/\omega,\mathbf{U},\hat{\mathbf{\Gamma}}^{(1)}}(\mathbf{x})$ can be evaluated as [6]

$$g_{\hat{\mathbf{X}}/\omega,\mathbf{U},\hat{\mathbf{\Gamma}}^{(1)}}(\mathbf{x}) = \frac{d}{d\mathbf{x}}^{(n)} \int_{\varphi(\mathbf{U},\boldsymbol{\lambda},\hat{\mathbf{\Gamma}}^{(1)},\boldsymbol{\gamma}^{(2)},\mathbf{y})<\mathbf{x}} \cdots \int g_{\mathbf{\Lambda},\mathbf{\Gamma}^{(2)},\mathbf{Y}/\omega,\mathbf{U},\hat{\mathbf{\Gamma}}^{(1)}}(\boldsymbol{\lambda},\boldsymbol{\gamma}^{(2)},\mathbf{y}) \cdot d\boldsymbol{\lambda}\, d\boldsymbol{\gamma}^{(2)} d\mathbf{y} \tag{5}$$

where $g_{\mathbf{\Lambda},\mathbf{\Gamma}^{(2)},\mathbf{Y}/\omega,\mathbf{U},\hat{\mathbf{\Gamma}}^{(1)}}(\boldsymbol{\lambda},\boldsymbol{\gamma}^{(2)},\mathbf{y})$ is the joint PDF of the vectors $\mathbf{\Lambda},\mathbf{\Gamma}^{(2)},\mathbf{Y}$ for a class ω and vectors $\mathbf{U},\hat{\mathbf{\Gamma}}^{(1)}$. Vectors $\boldsymbol{\lambda},\boldsymbol{\gamma}^{(2)},\mathbf{y}$ are the nonrandom analogs of the random vectors $\mathbf{\Lambda},\mathbf{\Gamma}^{(2)},\mathbf{Y}$, respectively.

It is noteworthy that the distribution of the object state vector $\mathbf{\Lambda}$ depends solely on the object class ω, that of the vector $\mathbf{\Gamma}^{(2)}$ bearing on the observation conditions depends solely on the estimate of the vector $\hat{\mathbf{\Gamma}}^{(1)}$, and that of the feature measurement error vector $\mathbf{Y}$ depends solely on the vector $\hat{\mathbf{\Gamma}}^{(1)}$ (primarily on the signal-to-noise ratio it contains). Also, the vectors $\mathbf{\Lambda}$, $\mathbf{\Gamma}$ and $\mathbf{Y}$ are independent. Hence, we may write

$$\begin{aligned} & g_{\mathbf{\Lambda},\mathbf{\Gamma}^{(2)},\mathbf{Y}/\omega,\mathbf{U},\hat{\mathbf{\Gamma}}^{(1)}}(\boldsymbol{\lambda},\boldsymbol{\gamma}^{(2)},\mathbf{y}) = \\ & = g_{\mathbf{\Lambda}/\omega,\mathbf{U}}(\boldsymbol{\lambda}) \cdot g_{\mathbf{\Gamma}^{(2)}/\hat{\mathbf{\Gamma}}^{(1)}}(\boldsymbol{\gamma}^{(2)}) \cdot g_{\mathbf{Y}/\hat{\mathbf{\Gamma}}^{(1)}}(\mathbf{y}) \end{aligned} \tag{6}$$

Insert Eq.(6) in Eq.(5) and suppose that the elements of the vector $\hat{\mathbf{\Gamma}}^{(1)}$ are measured with sufficient accuracy. Then, we finally get

$$g_{\hat{\mathbf{X}}/\omega,\mathbf{U},\hat{\mathbf{\Gamma}}^{(1)}}(\mathbf{x}) = \frac{d}{d\mathbf{x}} \int_{\varphi(\mathbf{U},\boldsymbol{\lambda},\hat{\mathbf{\Gamma}}^{(1)},\boldsymbol{\gamma}^{(2)},\mathbf{y})<\mathbf{x}} \cdots \int g_{\mathbf{\Lambda}/\omega,\mathbf{U}}(\boldsymbol{\lambda}) \cdot g_{\mathbf{\Gamma}^{(2)}/\hat{\mathbf{\Gamma}}^{(1)}}(\boldsymbol{\gamma}^{(2)}) \cdot g_{\mathbf{Y}/\hat{\mathbf{\Gamma}}^{(1)}}(\mathbf{y}) d\boldsymbol{\lambda}\, d\boldsymbol{\gamma}^{(2)} d\mathbf{y} \tag{7}$$

The main advantage of described approach is the correct account of the mutual dependence of vector $\hat{\mathbf{X}}$ elements.

The described approach is not free of difficulties. The main difficulty is that the likelihood function $g_{\hat{\mathbf{X}}/\omega,\mathbf{U}}(\hat{\mathbf{X}})$ in Eq.(1) as a rule has a several maximums with respect to ω and $\mathbf{U}$. Besides the amplitudes of the maximums are fluctuating during the target observation. That is why the choosing of the right maximum is the one more problem. The main method of its solving is tracking all the maximums with the purpose to determine the greatest and most firm maximum. Another difficulty of above approach is complicated resulting algorithm which require very powerful computer.

3 A practical example

To illustrate the reported approach to the problem of simultaneous classification and localization of underwater objects from their underwater sound fields, we consider a relatively simple example that is nevertheless of practical importance. Let one set the task of determining the class of the object and the distance to it at the moment of its detection. The determination should be based on the data obtained from a passive omnidirectional sonobuoy. Such formulation of the problem signifies that the only source of information for solving it is the measured noise spectrum of the object. This means that the vector $\hat{\mathbf{X}}$ of the estimated signal parameters used for solving the problem under study can be written in the form

$$\hat{\mathbf{X}} = \left\{ \hat{S}_1, \hat{S}_2, ..., \hat{S}_n \right\}, \tag{8}$$

where $\hat{S}_i$ is the estimate of the signal power in the *i*th frequency band (either narrow or wide) and n is the number of frequency bands.

Under certain assumptions, the stochastic model of the estimate (expressed in dB) has the form [7]

$$\hat{S}_i = P_0 + D_i(R,H,\Delta S_o) + Y_i, \qquad i=1,...,n, \tag{9}$$

where P_0 is the reduced target in noise dB (i.e., the acoustic pressure produced by the object as a monopole in some narrow frequency band Δf_o with the central frequency f_o at a small distance from the object R_o);

$D_i(R,H,\Delta S_o)$ is the loss (in dB) in the *i*th frequency band for a signal propagating from a source located at the depth H and at the distance R from the receiver and having the spectrum with the slope $\hat{S}_i$ at the point of radiation; Y_i is the measurement error for $\hat{S}_i$ in dB.

In model (Eq.(9)), the values P_0, R, H, ΔS_o, Y_i and the functions $D_i(R,H,\Delta S_o)$ ($i=1,...,n$) are random quantities. Taking into account that, among all them, only the functions $D_i(R,H,\Delta S_o)$ are mutually dependent, the PDF of the estimate of the vector $\hat{\mathbf{X}}$ can be written in a more simple form than that of Eq.(7):

$$g_{\hat{\mathbf{X}}/\omega,R,\hat{\mathbf{\Gamma}}^{(1)}}(x) = \int_{-\infty}^{\infty} g_{P_o/\omega}(p)dp \int_{0}^{\infty} g_{R/\omega,\hat{\mathbf{\Gamma}}^{(1)}}(r)dr \int_{0}^{\infty} g_{H/\omega,\hat{\mathbf{\Gamma}}^{(1)}}(h)dh \rightarrow$$

$$\rightarrow \int_{-\infty}^{\infty} g_{\Delta S_o/\omega}(s)ds \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} g_{D_1,\dots,D_n/r,h,s,\hat{\mathbf{\Gamma}}^{(1)}}(t_1,\dots,t_n) \times \prod_{i=1}^{n} g_{Y_i/\hat{\mathbf{\Gamma}}^{(1)}}(x_i - p - t_i)\, dt_1 \dots dt_n \qquad (10)$$

The PDF $g_{P_o/\omega}(p)$, $g_{\Delta S_o/\omega}(s)$, $g_{R/\omega,\hat{\mathbf{\Gamma}}^{(1)}}(r)$ and $g_{H/\omega,\hat{\mathbf{\Gamma}}^{(1)}}(h)$ on the right-hand side of Eq.(10) represent an *a priori* description of the noise spectrum and the coordinates of an object of every class ω with the current observation conditions characterized by the vector of the measured parameters $\hat{\mathbf{\Gamma}}^{(1)}$ being taken into account. The calculation of these PDF does not cause any difficulties. One cannot say the same about the calculation of the combined conditional PDF of the propagation loss for signals in different frequency bands $g_{D_1,\dots,D_n/r,h,s,\hat{\mathbf{\Gamma}}^{(1)}}(t_1,\dots,t_n)$. One of the approaches used for its calculation is described in [8]. The PDF of the error of signal power in the *i*th frequency band $g_{Y_i/\hat{\mathbf{\Gamma}}^{(1)}}(y_i)$ can be considered as normal with the zero mathematical expectation and the variance calculated by the formula given in [9].

As a result, in the case under study, algorithm Eq.(1) for the solution of the problem of simultaneous classification and determination of coordinates of underwater objects has the form

$$(\hat{\omega},\hat{R}) = arg \max_{\omega,R} g_{\hat{X}/\omega,R,\hat{\mathbf{\Gamma}}^{(1)}}(\hat{\mathbf{X}}) \quad , \qquad (11)$$

where the maximized PDF is calculated by Eq.(10).

The efficiency of this algorithm can be calculated by Eq.(2) and Eq.(3). In the case under consideration, the latter formula has the form

$$\sigma^2_{\hat{R}/\omega}(R) = \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} (\hat{R}_\omega(\mathbf{x}) - R)^2 g_{\hat{X}/\omega,R,\mathbf{\Gamma}^{(1)}}(\mathbf{x})d\mathbf{x} \qquad (12)$$

where $\sigma^2_{\hat{R}/\omega}(R)$ is the variance of the estimated distance $\hat{R}$ to the object of class ω as a function of the true distance R; $\hat{R}_\omega(\mathbf{x})$ is the estimated distance to the object of class ω for the vector of estimated parameters $\mathbf{x}$.

4 Conclusion

From the above, we can draw the following conclusions:

a) The proposed approach to solving the problems of simultaneous classification and localization of underwater objects from their underwater sound field takes into account the common physical basis of both constituent problems and treats the class of the object as one of its coordinates. This approach differs from the known ones in that it takes into account the statistic relationships between the parameters of the signal generated by the object of any recognizable class and the relationships between these parameters and the parameters characterizing the current observation conditions.

b) The key feature of the proposed approach to the problems of simultaneous classification and localization of underwater objects is the determination of the adequate distribution densities for the arguments of the stochastic models of signal parameters used in the calculations.

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